TRANSFORMATION OPTICS: DESIGNING OPTICS ON THE NANOSCALE

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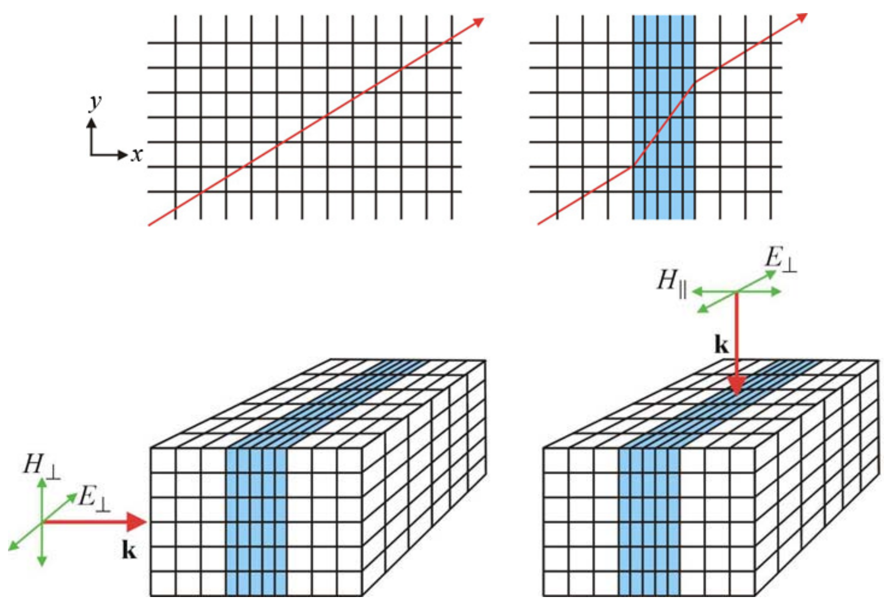
For design of conventional optical devices such as camera lenses we turn to ray tracing programs. Trajectories of rays through a lens give a simple and intuitive appreciation of device performance as well as precise parameters for construction. In contrast optics on the nanoscale is almost exclusively concerned with the near field where rays have no meaning. The complex patterns of resonances seen in metallic nano objects seem to arise in a confusing and counter intuitive fashion. In this talk I shall describe an alternative approach to optical design based on coordinate transformations. The underlying theory is based on Maxwell's equations not on the ray approximation and therefore can be applied to both near and far fields.

The concept is a straightforward one: we start with a simple configuration of fields: a ray propagating in free space, or the magnetic field around a dipole magnet, for example, and then distort the fields until the rays or the field lines follow the trajectories we wish to see. If we imagine that the original fields were embedded in an elastic matrix on which a set of coordinates was inscribed, then the distortion could be described mathematically as a transformation from the old to the new set of distorted coordinates. We can then use some mathematics to discover what values of ε, μ would ensure that Maxwell's equations are obeyed for the desired configuration of fields

To give a flavour of how the scheme operates imagine the simplest possible distortion of space in which a section of the $x-$ axis is compressed.

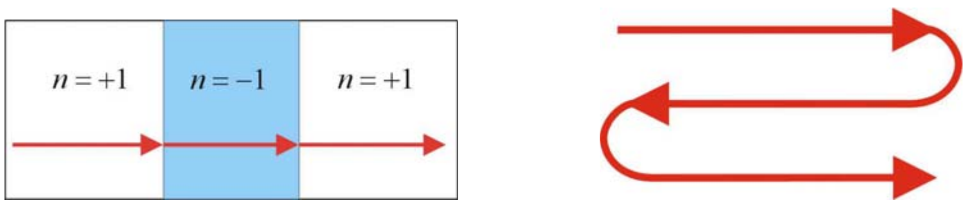


Top: a simple coordinate transformation that compresses a section of the $x-$ axis. As a result rays follow a distorted trajectory shown on the top right but emerge from the compressed region travelling in exactly the same direction with the same phase as before. Bottom: requiring that a ray pass through the compressed region with the same phase change as through uncompressed space enables us to predict the metamaterial properties that would realise this trajectory for a ray.

If m is the compression factor, and ε_y and μ_y are the components of the respective tensors perpendicular to the x – axis, then we deduce that,

$$\varepsilon_y = \mu_y = m^{-1}$$
$$\varepsilon_x = \mu_x = m$$

Transformation optics can also be used to give another interpretation to the Veselago lens shown below.



Left: in the x, y, z coordinate system space is single valued and a ray progresses continuously through the region of negative refraction. Right: an equally legitimate view point is that the refractive index is everywhere positive but space is triple valued, doubling back on itself so that each point within the range of the lens is crossed thrice.

The lens exists in a coordinate system x, y, z but viewed from outside it would appear as though the region between the object plane and the image plane vanishes. This can be expressed as a coordinate transformation,

$$\begin{aligned}
&y' = y, \quad z' = z \\
&x' = x, && x < x_1, \\
&x' = (2x_1 - x), && x_1 < x < x_1 + d, \\
&x' = x - 2d, && x_1 + d < x
\end{aligned}$$

where the lens is assumed lies in the range $x_1 < x < x_1 + d$ where the x, y, z space maps on to a triple valued x', y', z' space. Applying the transformation procedure gives,

$$\begin{aligned}
&\varepsilon = \mu = +1, && x < x_1, \\
&\varepsilon = \mu = -1, && x_1 < x < x_1 + d, \\
&\varepsilon = \mu = +1, && x_1 + d < x
\end{aligned}$$

Now we can give a geometrical interpretation to the lens: it comprises a section of 'negative' space that annihilates an equivalent thickness of vacuum.

[1] JB Pendry, D Schurig, and DR Smith, *Science*, **312** (2006) 1780-2.