Poster

Donor-bound electrons in quantum rings under magnetic fields

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Recent advances in nanofabrication of quantum devices enables to study electronic properties of quantum rings (QRs) in a very controllable way [1–3]. QRs are small semiconductor ring-shape structures in which electrons are confined in all spatial dimensions so as a consequence, QRs are very promising systems due to their physical properties as well as their potential application in electronic devices.

Donor or acceptor binding energies are also modified due to presence of a QR as compared to bulk semiconductors. Therefore, binding energy carries information about the confinement properties of electrons and holes in QRs. Consequently, spectroscopy studies of radiative recombinations of donor-bound electrons or acceptor-bound holes provide a unique tool to characterize the electronic properties of QRs.

In this work, we consider on-center and off-center donors in a two-dimensional (2D) QR within the effective-mass approximation. The QR is assumed to be of finite width, and a strong magnetic field is applied perpendicular to the plane of the QR. The binding energy of a single electron bound to an on-center donor is obtained by exact diagonalization of the radial Hamiltonian. In the case of off-center impurities, the standard perturbation approach allows us to calculate the binding energy as a function of the impurity position and magnetic field. As a major result, we find a strong dependence of the confining properties of the QR upon the magnetic field. In particular, an abrupt transition of the localization properties of the electronic envelope-function is observed at a critical magnetic field.

We will focus on electron states close to the bottom of the conduction-band and neglect nonparabolicity effects hereafter. Then, a one-band effective-mass Hamiltonian suffices to obtain accurate results. For simplicity, we assume the same effective-mass m^* at the Γ valley in both semiconductors, namely inside and outside the QR.

The radial function $R(\rho)$ satisfies the following effective-mass equation

$$\left[-\frac{\hbar^2}{2m^*}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}\right)+\frac{\hbar^2}{2m^*}\frac{\ell^2}{\rho^2}+\frac{m}{8}\,\omega_c^2\rho^2+V(\rho)\right]R(\rho)=\left(E-\frac{\ell}{2}\,\hbar\omega_c\right)R(\rho)\ , \tag{1}$$

where the cyclotron frequency is $\omega_c = eB/m^*c$. The conduction band-edge profile along the radial direction of the QR is given by

$$V(\rho)=\begin{cases} 0\ , & \rho_1<\rho<\rho_2\ , \\ V_0\ , & \text{otherwise}\ , \end{cases} \tag{2}$$

with $V_0 > 0$. Here ρ_1 and ρ_2 are the inner and outer radii of the QR, respectively. Solution of Eq. (1) is given in Ref. [4].

Let us now introduce a single ionized donor at the center of the QR

$$V_C(\rho) = -\frac{e^2}{\epsilon_r\rho}\ , \tag{3}$$

ϵ_r being the relative dielectric constant of the semiconductors. In this case, the axial symmetry is not broken and we obtain an equation similar to Eq. (1), with an addional Coulomb term. We solved it numerically using dimensionless coordinates and a standard finite-difference approach.

As we can see in the following figures an electronic tunneling-transition between the well and the donor-bound states occur at critical magnetic fields values regarding to the electronic state of the electron. In the Fig 1 the $|10>$ (where the first quantum number is n and the second one is $|l| = 0...n-1$ the angular momentum) delocalization occurs at $B \sim 20T$ while in the Fig 2, the electronic state is $|20>$ and the delocalization occurs at lower magnetic fields. This behaviour is related to alignment between the energy levels of the QR and the imputity since the variation of a bulk level (impurity) is slower than the well-one Fig 3. In those figure two anicrossing points could be observed at the delocalization magnetic fields of the $|10>$ and $|20>$ states.

We have also consider the case of one off-center donor located at a distance $d = |\boldsymbol{\rho}_i|$ of the center, $\boldsymbol{\rho}_i$ being its position in the plane of the QR. Clearly the axial symmetry is broken and an equation similar to Eq. (1) does not

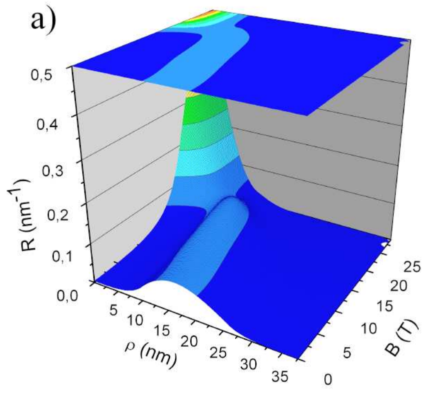

FIG. 1: Envelope function for $|10>$ versus applied magnetic field for a QR with an on-center impurity

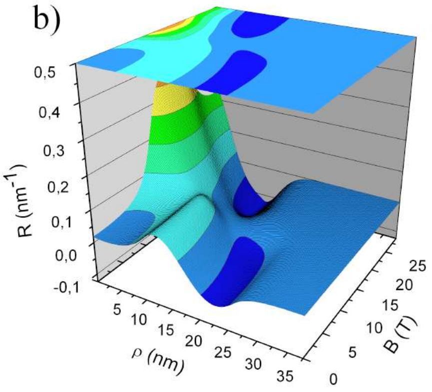

FIG. 2: Envelope function for $|20>$ versus applied magnetic field for a QR with an on-center impurity.

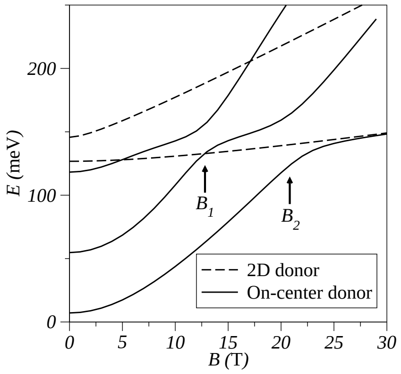

FIG. 3: Energy variation versus perpendicular applied magnetic field for the donor and the QR electron levels. Two anticrossing points between QR levels can be observed

hold. We are then faced to a 2D effective-mass equation for the complete $\chi(\boldsymbol{\rho})$ envelope-function that have been solved analitically in function of the distance to QR origin using the following equations

$$\langle \ell \,|\, V_C(\boldsymbol{\rho}) \,|\, \ell' \rangle = -\frac{e^2}{2\pi\epsilon_r} \sum_{m=0}^{\infty} A_m(\Delta\ell) \int_0^{\infty} d\rho \, \rho f_m(\rho) R_\ell(\rho) R_{\ell'}(\rho) \ , \tag{4a}$$

where $R_\ell(\rho)$ is the solution of Eq. (1), $\Delta\ell = \ell' - \ell$ and

$$A_m(\Delta\ell) = \int_0^{2\pi} d\phi \, e^{i\Delta\ell\phi} P_m(\cos\phi) \ . \tag{4b}$$

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