



# Pressure-Induced Structural Transitions in Multiwall Carbon Nanotubes

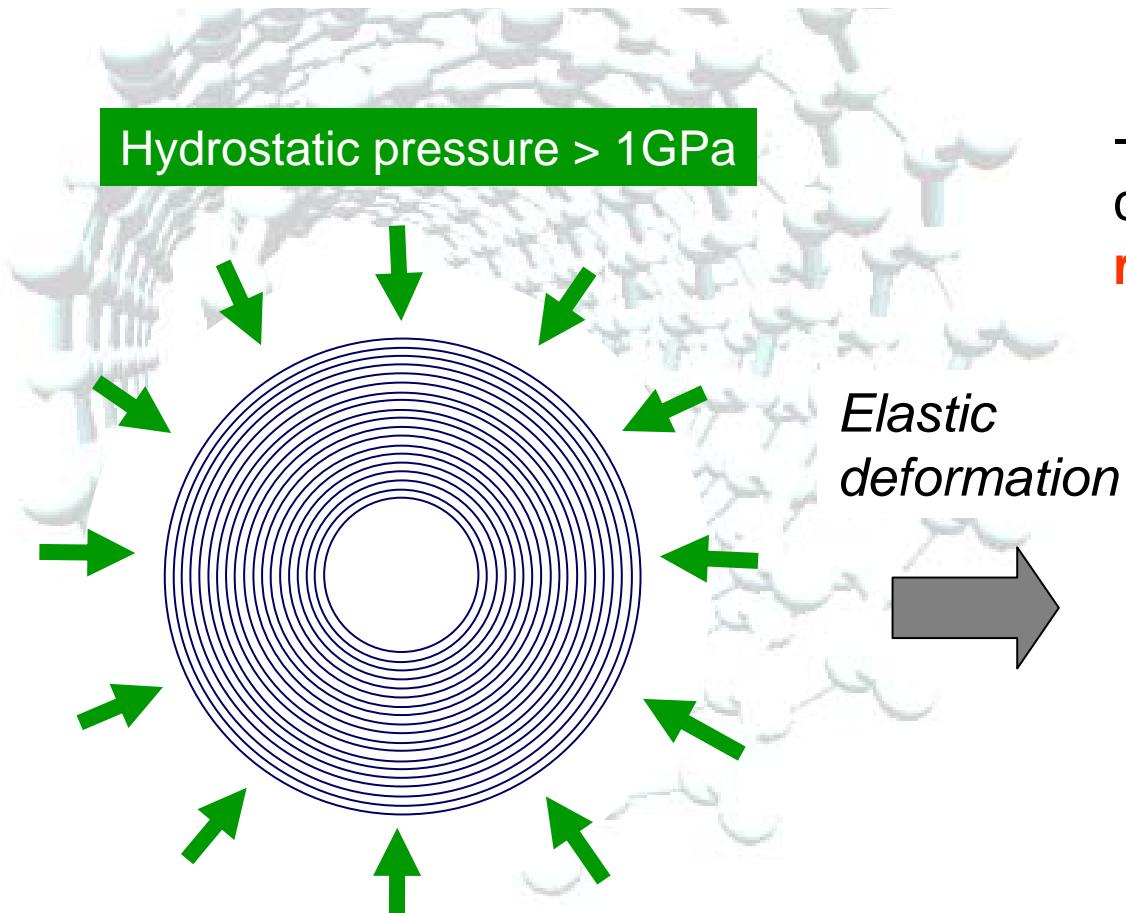
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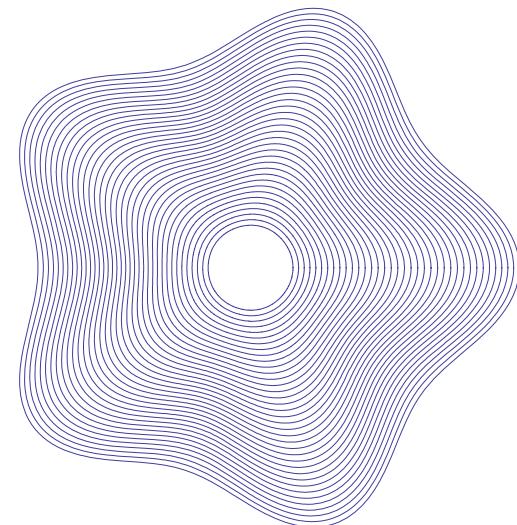
See H. Shima and M. Sato, *Nanotechnology* *in press*

# 0. Main Finding

= Pressure-induced **Radial Corrugation** of MWNT



- The cross-sectional shape changes from circular to **radially corrugated** one.

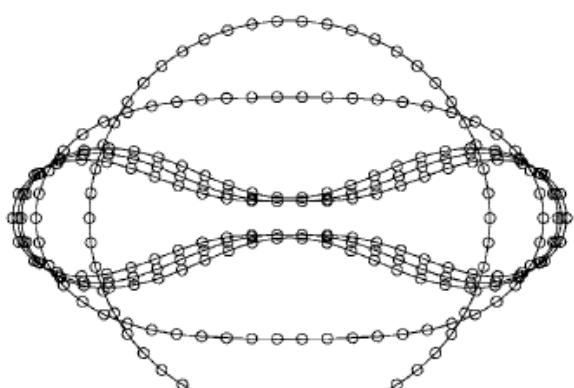


# 1. Radial collapse of SWNTs

Carbon nanotubes are

- extraordinarily **stiff** in the **axial** direction,  
but
- highly **flexible** in the **radial** direction.

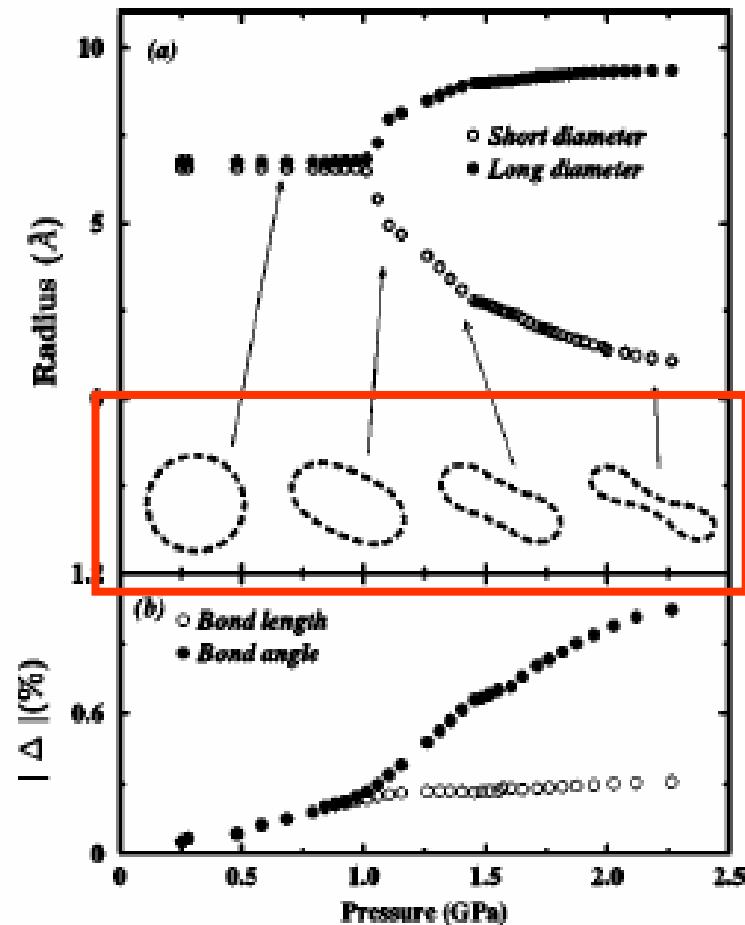
→ **Radial collapse** occurs at 1.0GPa



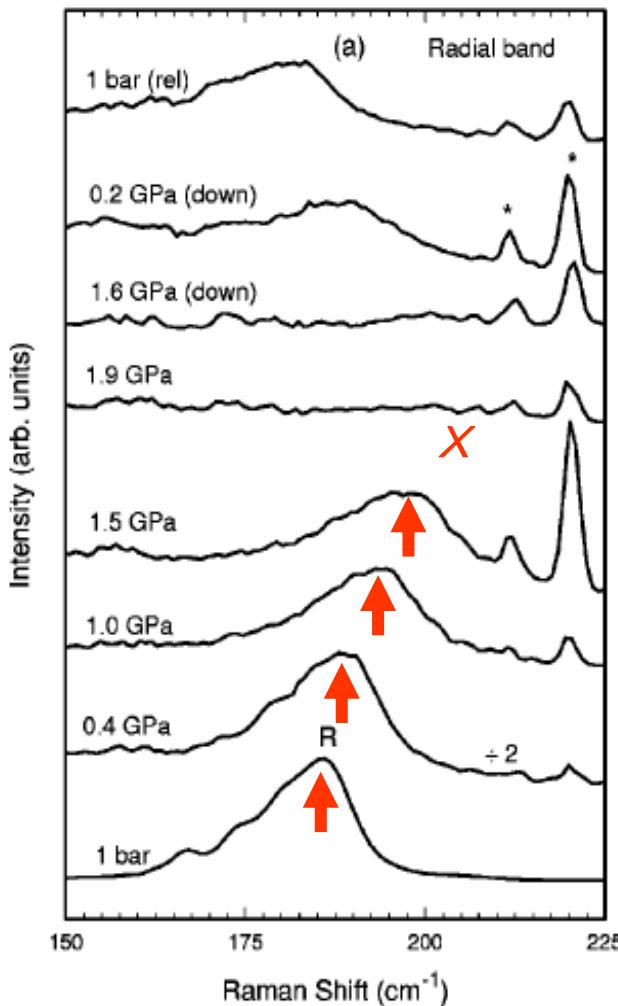
S. Zhang *et al.*, PRB 73 (2006) 075423.

Results of MD simulations 16

Sun *et al.*, PRB 70 (2004) 165417



# 1. Radial collapse of SWNTs



Raman spectroscopy:

Vanishing a radial breathing mode

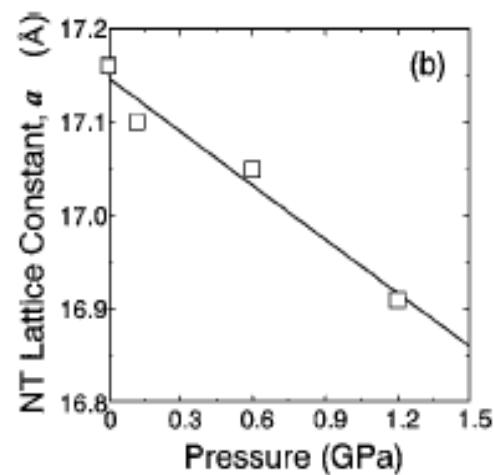
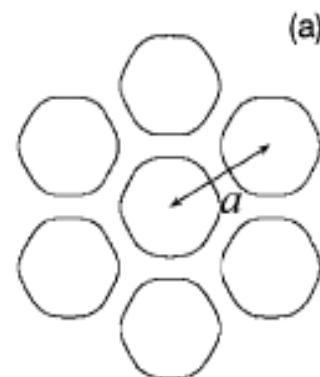
Venkateswaran *et al.*, PRB 59(1999) 10928



X-ray diffraction:

Polygonization of SWNT-bundle

Tang *et al.*, PRL 85 (2000) 1887



## 2. Motivation

What takes place in Multi-walled nanotubes?

Structural features of multi-walled carbon nanotubes:

- Multiple concentric walls **interact with each other** through the intermolecular forces.
- External pressure leads to a **mechanical instability in outside walls** due to their large tube diameters.
- **Inner walls** are relatively stiff in the radial direction so that they can push back the surrounding outer walls.

Atomic-scale simulations for MWNTs = Very challenging!

→ Alternative approach: **Continuum elastic approximation**

### 3. Model and Method

Continuum elastic-shell model for MWNT

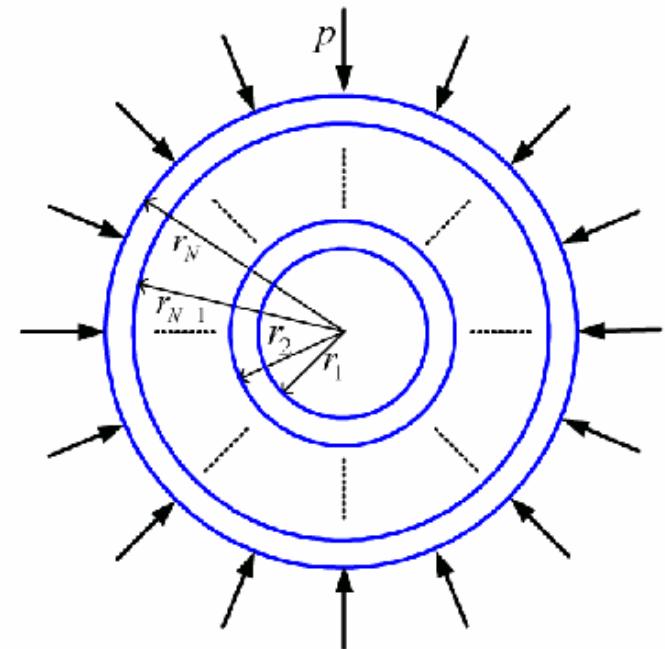
Ru, PRB **62** (2000) 16962

Sudak JAP **94** (2003) 7281

Leung PRB **71** (2005) 165415

He, J. Mech. Phys. Solids **53** (2005) 303

Wang JAP **99** (2006) 114317

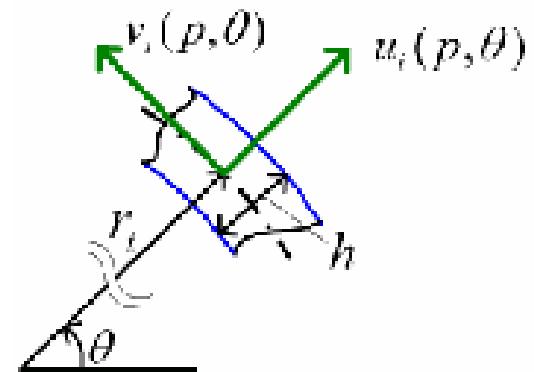


● **Displacements** of a surface element:

radial  $u(\theta, p)$

circumferential  $v(\theta, p)$

→ Obtaining the **mechanical energy** of the deformed MWNT, and then its stable cross-sectional shape under high hydrostatic pressure

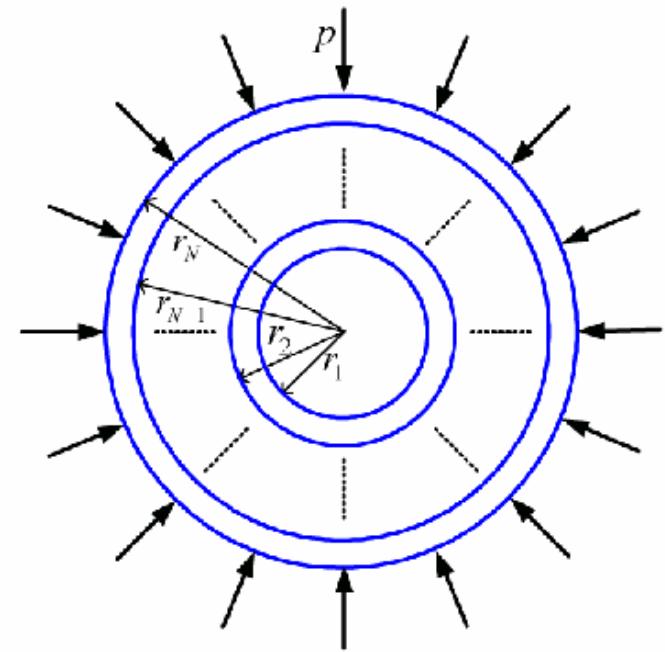


### 3. Model and Method

The mechanical energy of MWNT:

$$U[p, u_i(p), v_i(p)] = U_D + U_I + \Omega$$

→ Apply a **variational method** to  $U$  in terms of  $u_i, v_i$  in order to determine the **stable cross-section** under high pressure  $p$



Deformation energy: 
$$U_D = \sum_{i=1}^N \int_0^{2\pi} \left\{ \frac{Eh}{2(1-\nu^2)r_i} \left[ u_i + v_i' + \frac{(u_i' - v_i)^2}{2r_i} \right]^2 + \frac{Eh^3}{24(1-\nu^2)} \frac{(u_i'' - v_i')^2}{r_i^3} \right\} d\theta$$

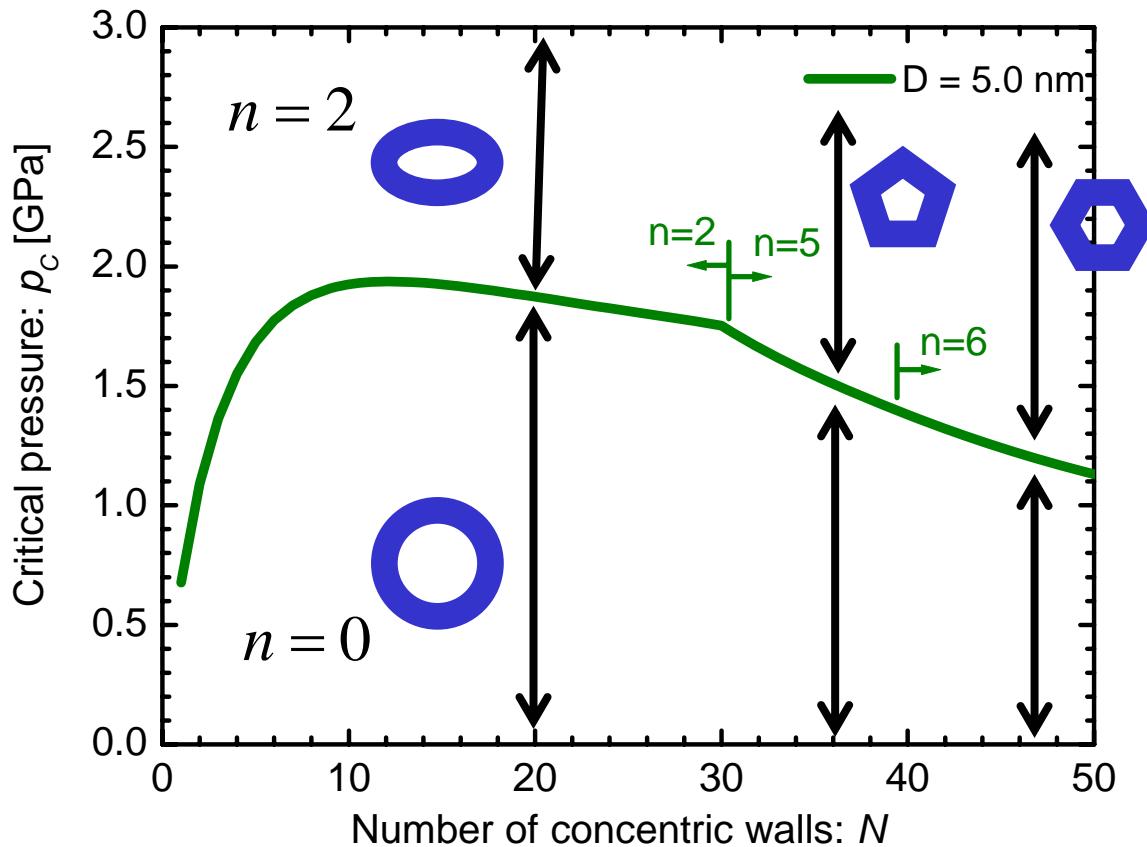
Inter-wall vdW energy 
$$U_I = \sum_{i=1}^{N-1} \frac{c_{i,i+1}r_i}{2} \int_0^{2\pi} (u_i - u_{i+1})^2 d\theta + \sum_{i=2}^N \frac{c_{i,i-1}r_i}{2} \int_0^{2\pi} (u_i - u_{i-1})^2 d\theta$$

Pressure-induced energy 
$$\Omega = p \int_0^{2\pi} \left( r_N u_N + \frac{u_N^2 + v_N^2 - u_N' v_N + u_N v_N'}{2} \right) d\theta$$

See H.Shima and M.Sato, *Nanotechnology in press*

## 4. Results

### [1] Critical pressure curves



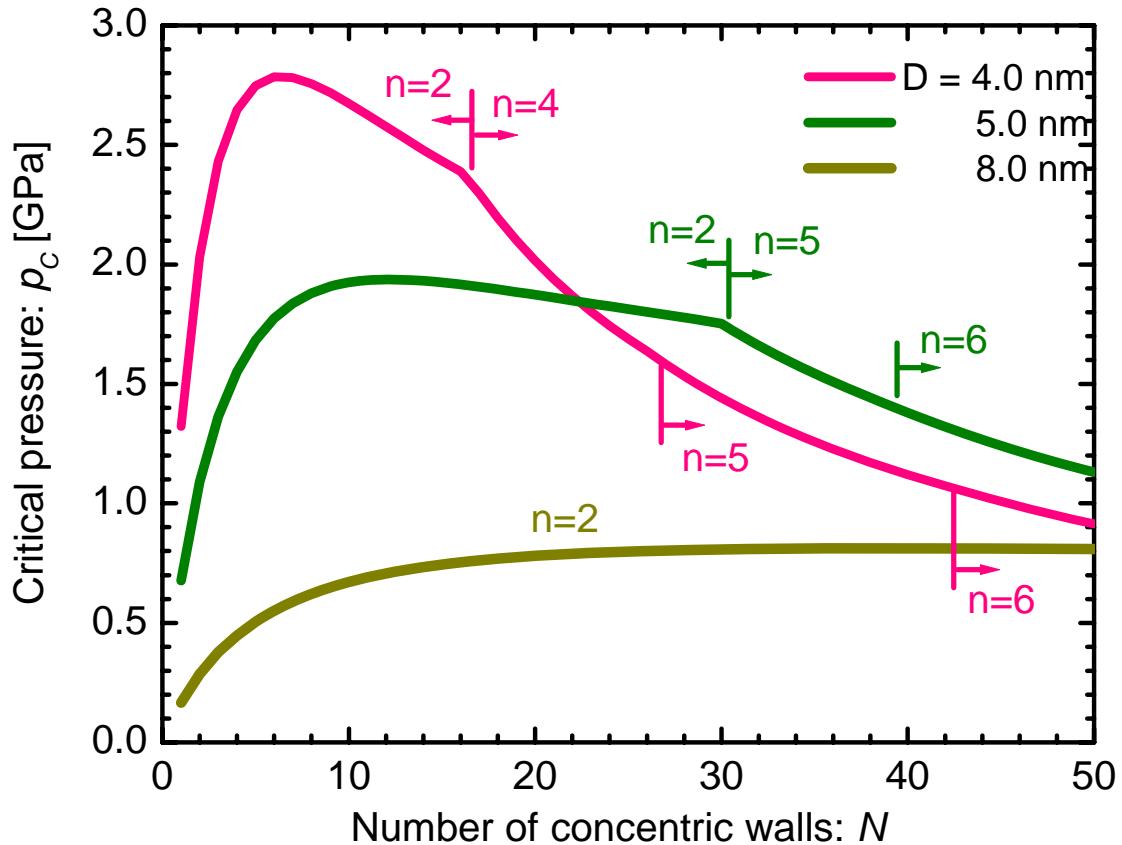
For given  $N$  and  $D$ , radial deformation occurs just above  $p_c$

When  $N$  exceeds 30, radial corrugation is observed

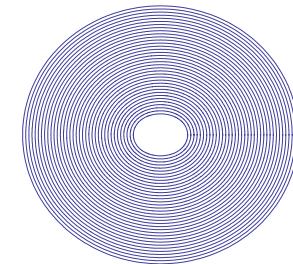
The wave number  $n$  of corrugation mode depends on  $N$  and  $D$

## 4. Results

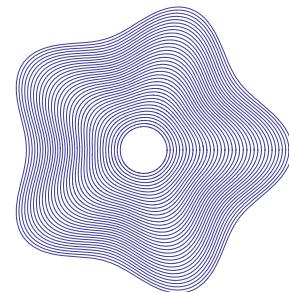
### [1] Critical pressure curves



$n = 2$  : Elliptic mode



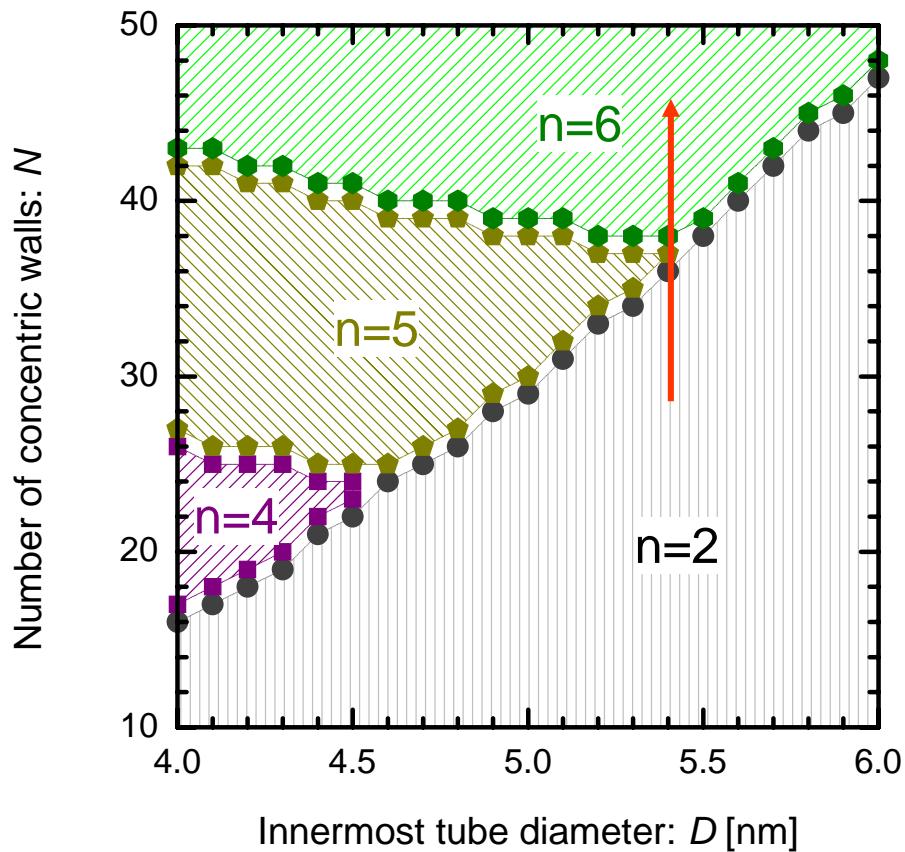
$n = 5$  : Corrugation mode



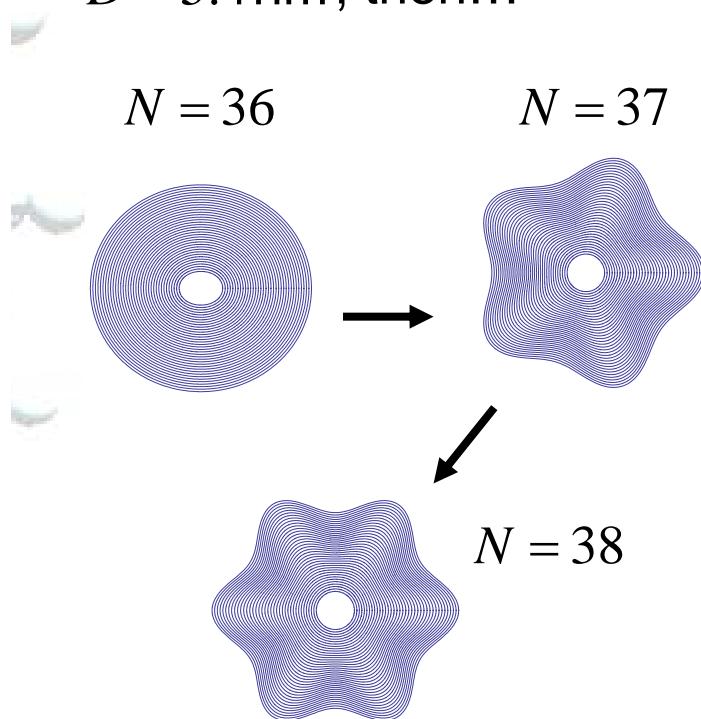
(= peculiar to MWNT with  $N \gg 1$  )

## 4. Results

### [2] Phase diagram



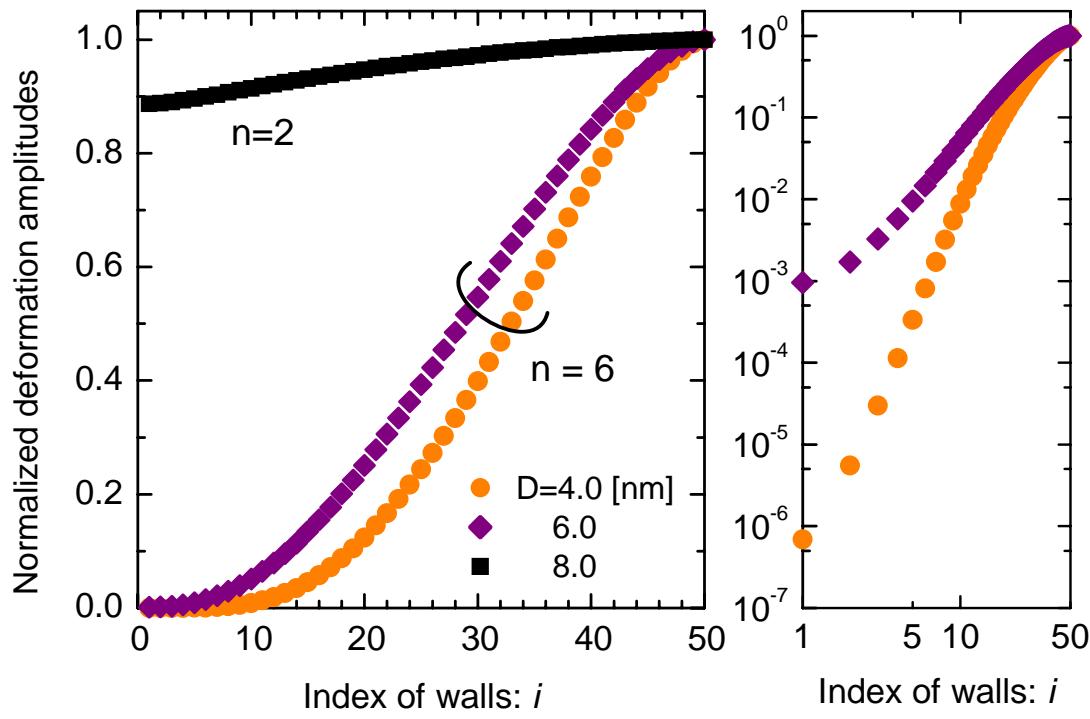
Increasing  $N$  with fixed  $D = 5.4\text{ nm}$ , then...



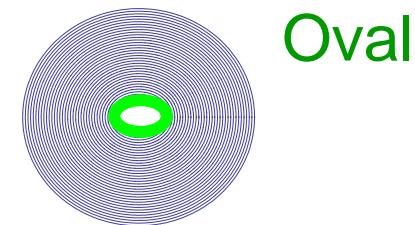
## 4. Results

### [3] Geometric persistence of the innermost tube

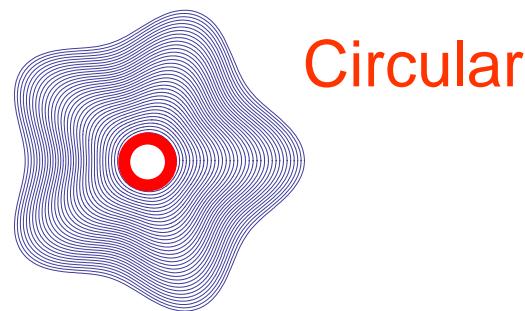
*In all corrugation modes, the **circular shape** of the **innermost tube** persists even under high pressure.*



$n = 2$  : Elliptic mode



$n = 5$  : Corrugation mode



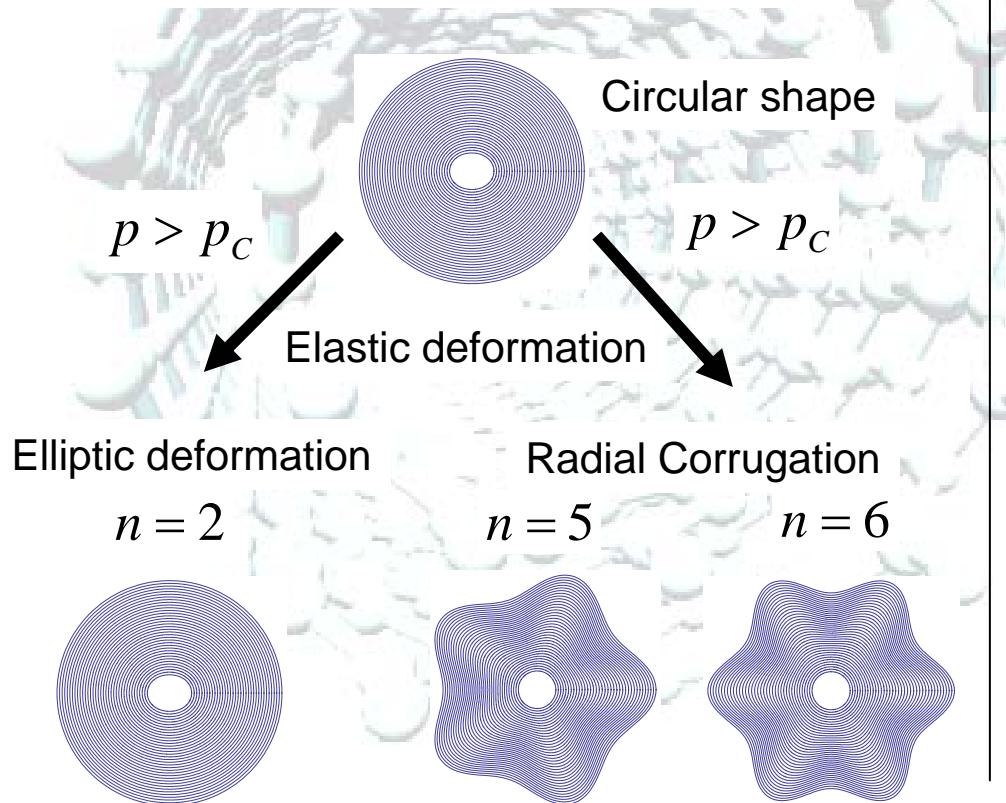
# 5. Summary

(Contact to: [shima@eng.hokudai.ac.jp](mailto:shima@eng.hokudai.ac.jp))



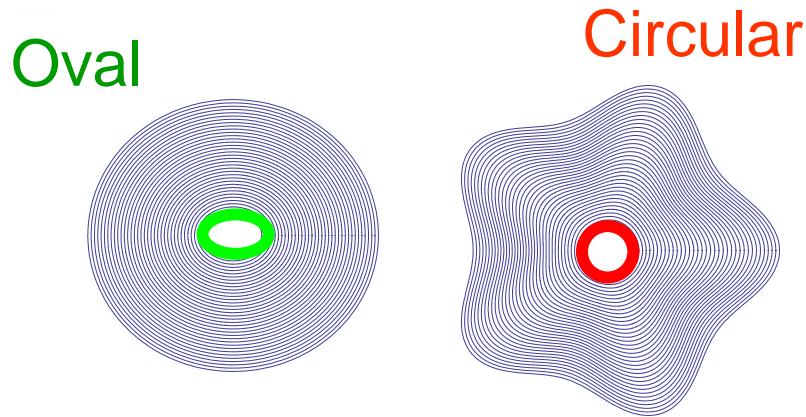
The main findings of this study are:

(1) Pressure-induced radial corrugation  
in the cross-section of MWNT



(2) Mode index  $n$  depend on:  
i) the tube diameter  $D$  and  
ii) the number of concentric wall  $N$

(3) Persistent cylindrical geometry  
of the innermost tube of MWNT



# 5. Summary

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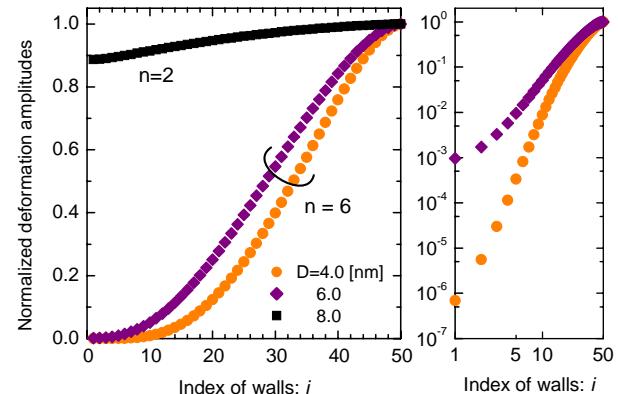
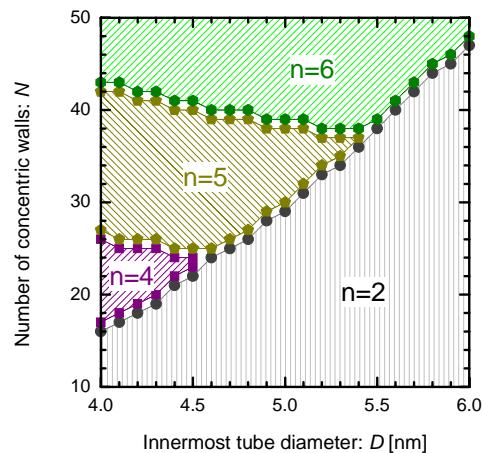
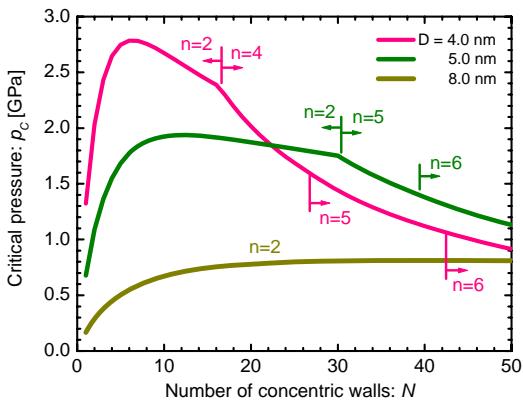


For multi-walled carbon nanotubes, we have demonstrated ...

(1) Critical pressure curves

(2) Phase diagram of radial deformation

(3) Geometric persistence of the innermost tube



See H.Shima and M.Sato, *Nanotechnology* in press