ARTIFICIAL FEW-ELECTRON SINGLE AND MOLECULAR QUANTUM DOTS IN LOW MAGNETIC FIELDS: ELECTRONIC SPECTRA, SPIN CONFIGURATIONS, AND HEISENBERG CLUSTERS

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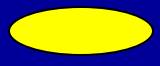
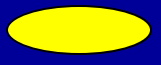
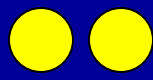
N=2e: C. Ellenberger et al, Phys. Rev. Lett. **96,** 126806 (2006),
T. Ihn et al., Int. J. Mod. Phys. B **21,** 1316 (2007) *(single anisotropic dots)*



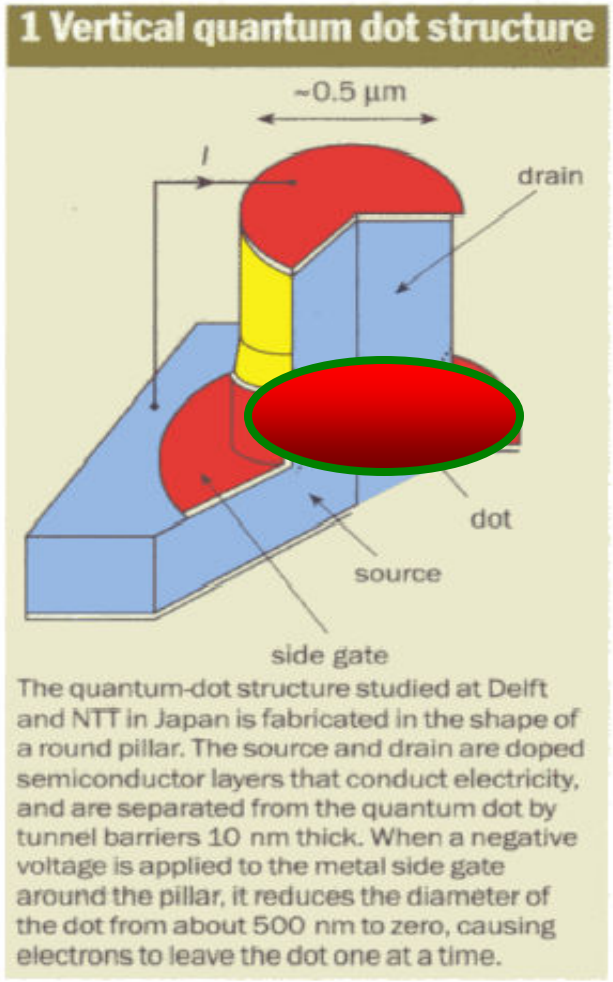
N=3e: Yuesong Li et al.: PRB **76**, 245310 (2007) *(single anisotropic dots)*



N=4e: Ying Li et al.: PRB **80,** 045326 (2009) (double quantum dots)



Method: Exact Diagonalization (EXD)



Vertical QD (Delft)

Electrostatic confinement

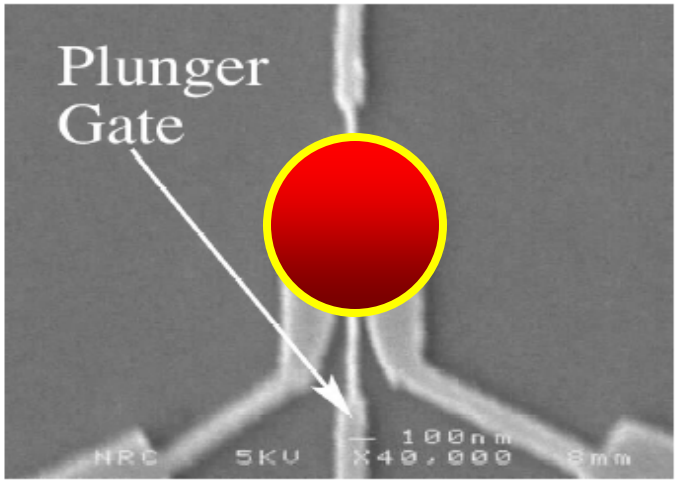
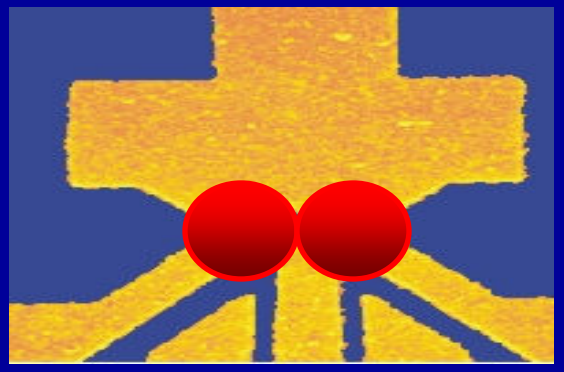


FIG. 1. SEM image of the gate geometry forming the quantum dot. This geometry enables a precisely known number of electrons ($N=0,1,2,\ldots,50$) to be trapped (Ref. 13) and produces a quasiparabolic confinement potential. Sweeping the plunger-gate voltage tunes both the shape and the chemical potential of the quantum dot.

Lateral QD (Ottawa)



Lateral QD Molecule (Delft)

HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$\mathcal{H} = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i} \frac{e^2}{\kappa r_{ij}}$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^{\,2}}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

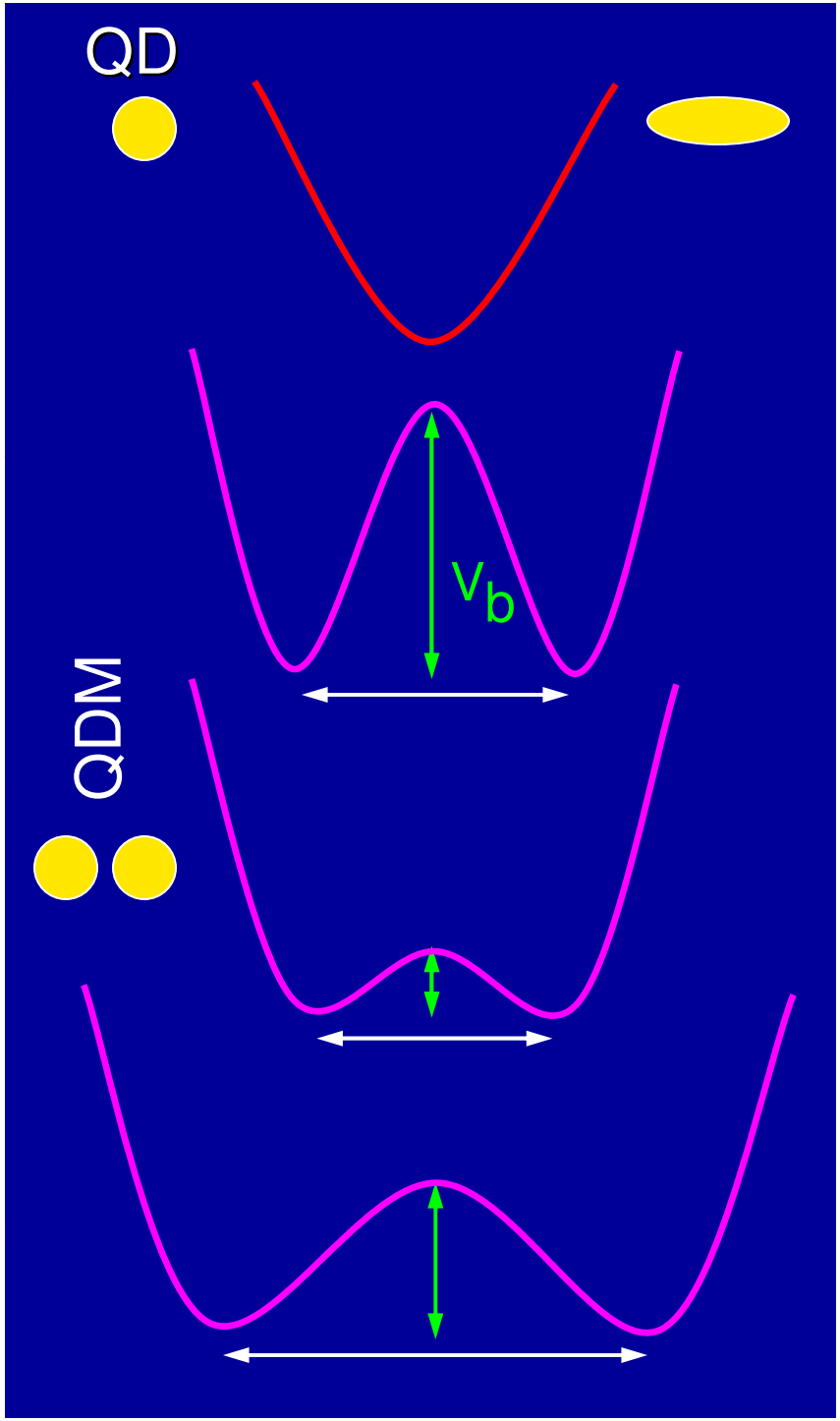
Two-center oscillator with V_b control } QDM

$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^{\,2}]/2m^* + g^* \mu_B \vec{B}\cdot\vec{S}_i/\hbar$$

$$\vec{A}_i = B(-y_i, x_i, 0)/2$$

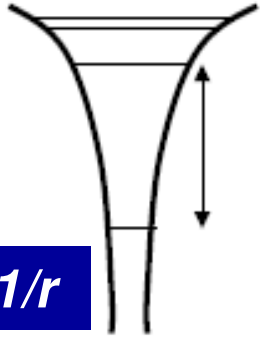
Zeeman

$\mathcal{H}$ can be generalized to:
Multi-component systems

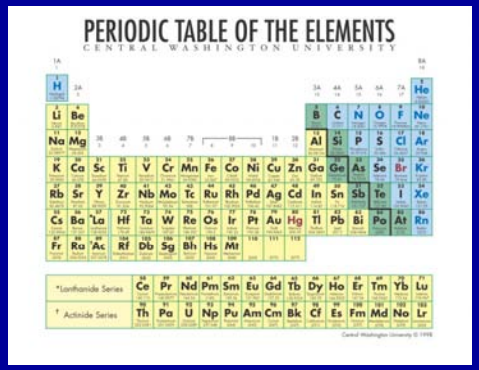


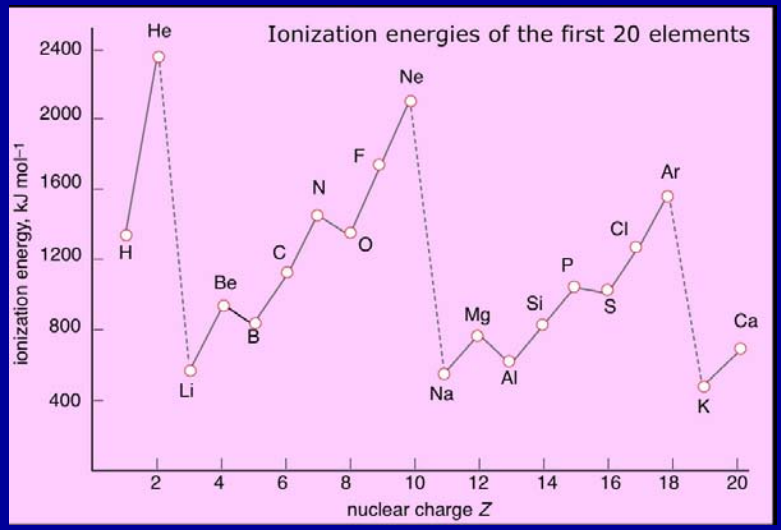
Central potential: Electronic shell effects

Natural atoms



V(r) ~ 1/r



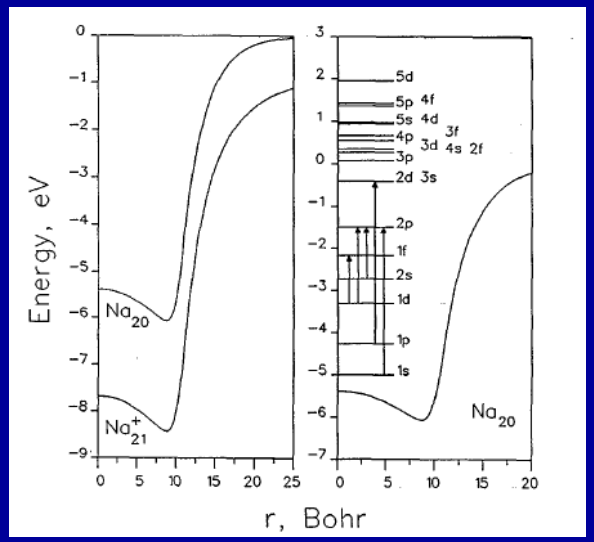


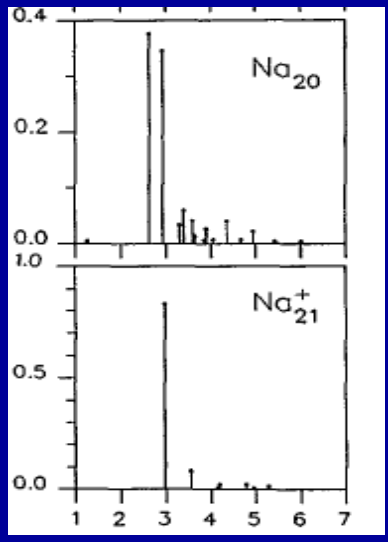
3D Clusters/Superatom: Density functional theory

Magic numbers: 2, 8, 18, 20, …

Giant resonances/
optical response/ RPA

Yannouleas et al., PRL **63**, 255 (1989)





Wigner Crystals

DECEMBER 1, 1934 PHYSICAL REVIEW VOLUME 46

On the Interaction of Electrons in Metals

E. WIGNER, *Princeton University*

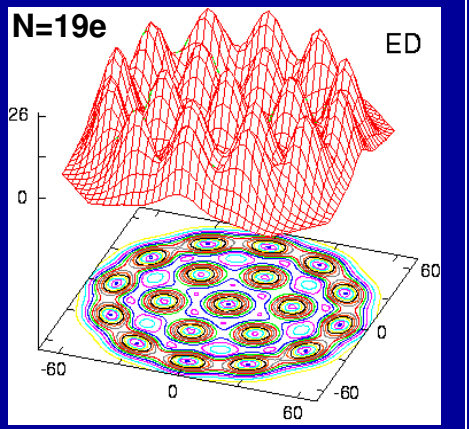
(Received October 15, 1934)

The energy of interaction between free electrons in an electron gas is considered. The interaction energy of electrons with parallel spin is known to be that of the space charges plus the exchange integrals, and these terms modify the shape of the wave functions but slightly. The

fact that the electrons repell each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function. In the present paper it is attempted to calculate this "correlation energy" by an approximation

… electrons repell each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function.
… "correlation energy" …

Wigner molecule in a 2D circular QD.
Electron density (ED) from
Unrestricted Hartree-Fock.
Symmetry breaking (localized orbitals).
Concentric rings (1,6,12).



CONTROLLING PARAMETERS

IN SINGLE QD'S: WIGNER CRYSTALLIZATION

- Essential Parameter at B=0: (parabolic confinement)

$$R_W = (e^2/\kappa l_0)/\hbar\omega_0 \sim 1/(\hbar^3\omega_0)^{1/2}$$

e-e Coulomb repulsion — kinetic energy

$$l_0 = (\hbar/m^*\omega_0)^{1/2}$$ } Spatial Extent of 1s s.p. state

κ : dielectric const. (12.9)

m^* : e effective mass ($0.067\ m_e$) GaAS

$\hbar\omega_0$ (5 - 1 meV) => R_W (1.48 - 3.31)

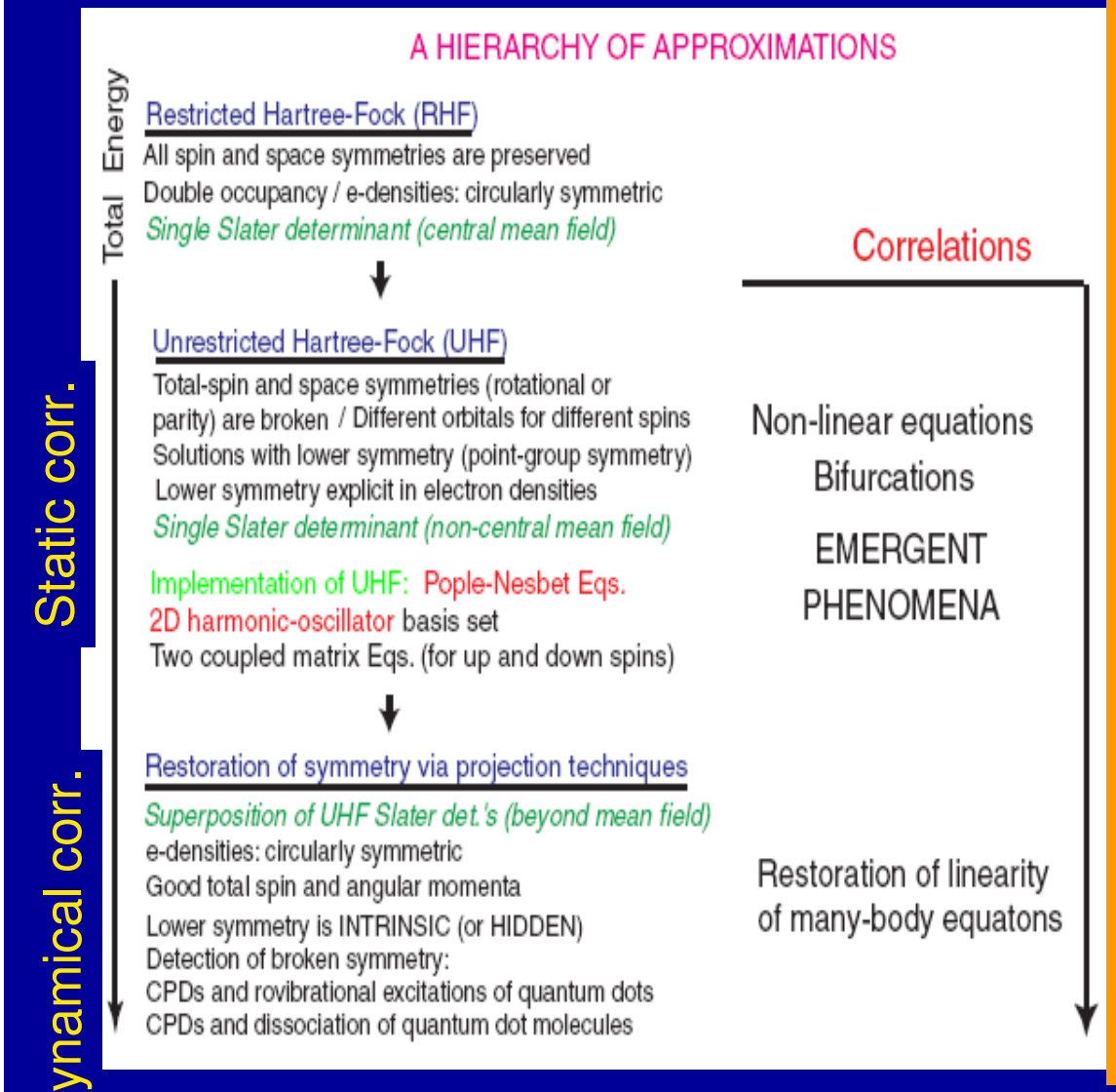
- In a magnetic field, essential parameter is B itself

IN QDM'S: DISSOCIATION (Electron puddles, Mott transition)

Essential parameters: Separation (d)
Potential barrier (V_b)
Magnetic field (B)

WAVE-FUNCTION BASED APPROACHES

TWO-STEP METHOD



EXACT DIAGONALIZATION

When possible (small N): High numerical accuracy

Physics less transparent compared to "THE TWO-STEP"

Pair correlation functions, CPDs

Yannouleas and Landman, Rep. Prog. Phys. ***70****, 2067 (2007)*

Applications of EXD approach
(to strongly-correlated 2D electrostatic QDs)

1) Detailed description of excitation spectra (advantage over DFT, etc…)

2) Description of many-body entanglement (advantage over DFT, etc…)

3) Transport properties in QDs (current intensity, phase lapses in Aharonov-Bohm interferometry)

EXD many-body wave function:

$$|\Psi_N^{\mathrm{EXD}}(S,S_z;k)\rangle = \sum_I C_I^N(S,S_z;k)|SD(I;N,S_z)\rangle$$

I ~ 500,000

Slater determinant

All symmetries conserved: total L, total S, S_z

Excitation spectrum of (elliptic) Anisotropic Quantum Dot Helium (Pinned WM)

C. Ellenberger et al., Phys. Rev. Lett. **96**, 126806 (2006)

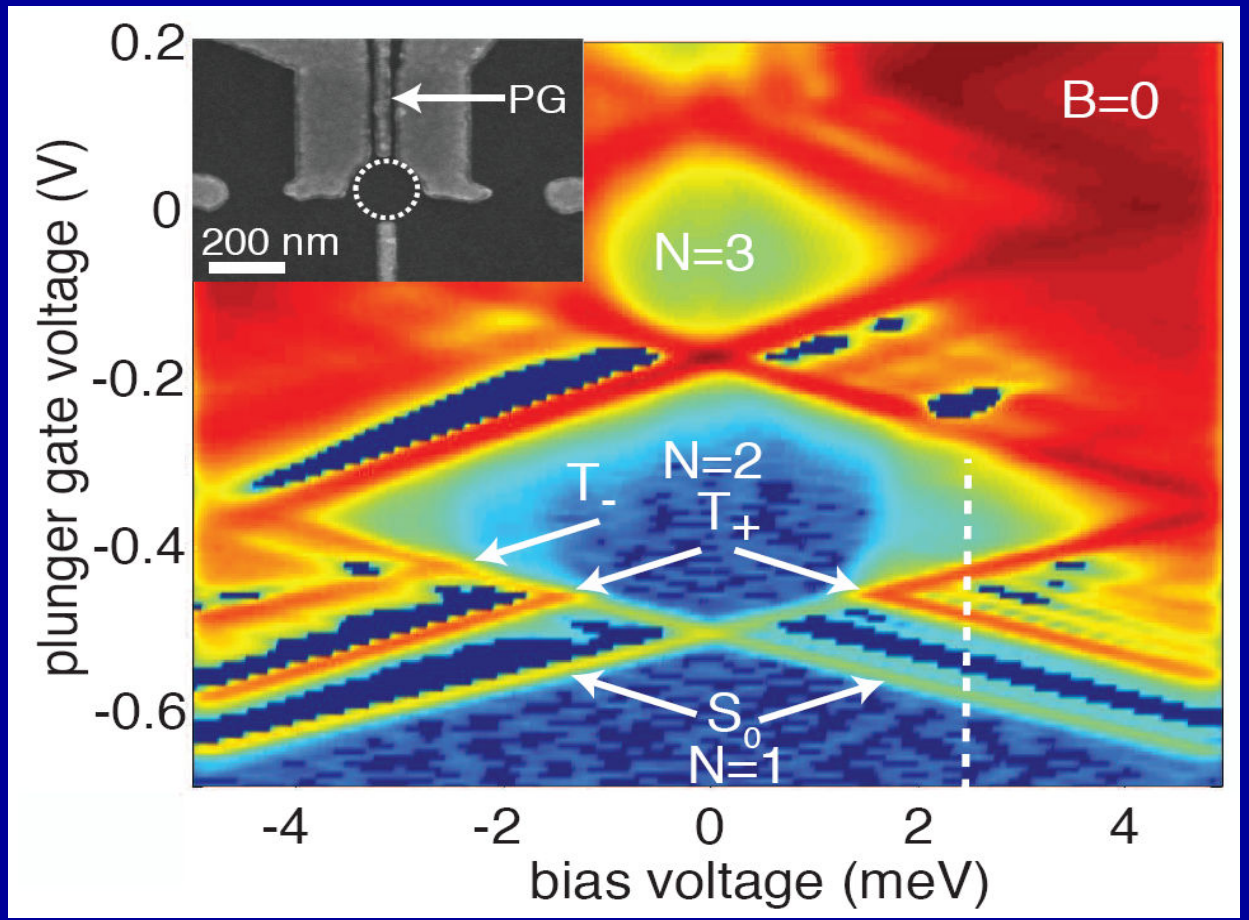
(No Zeeman splitting)

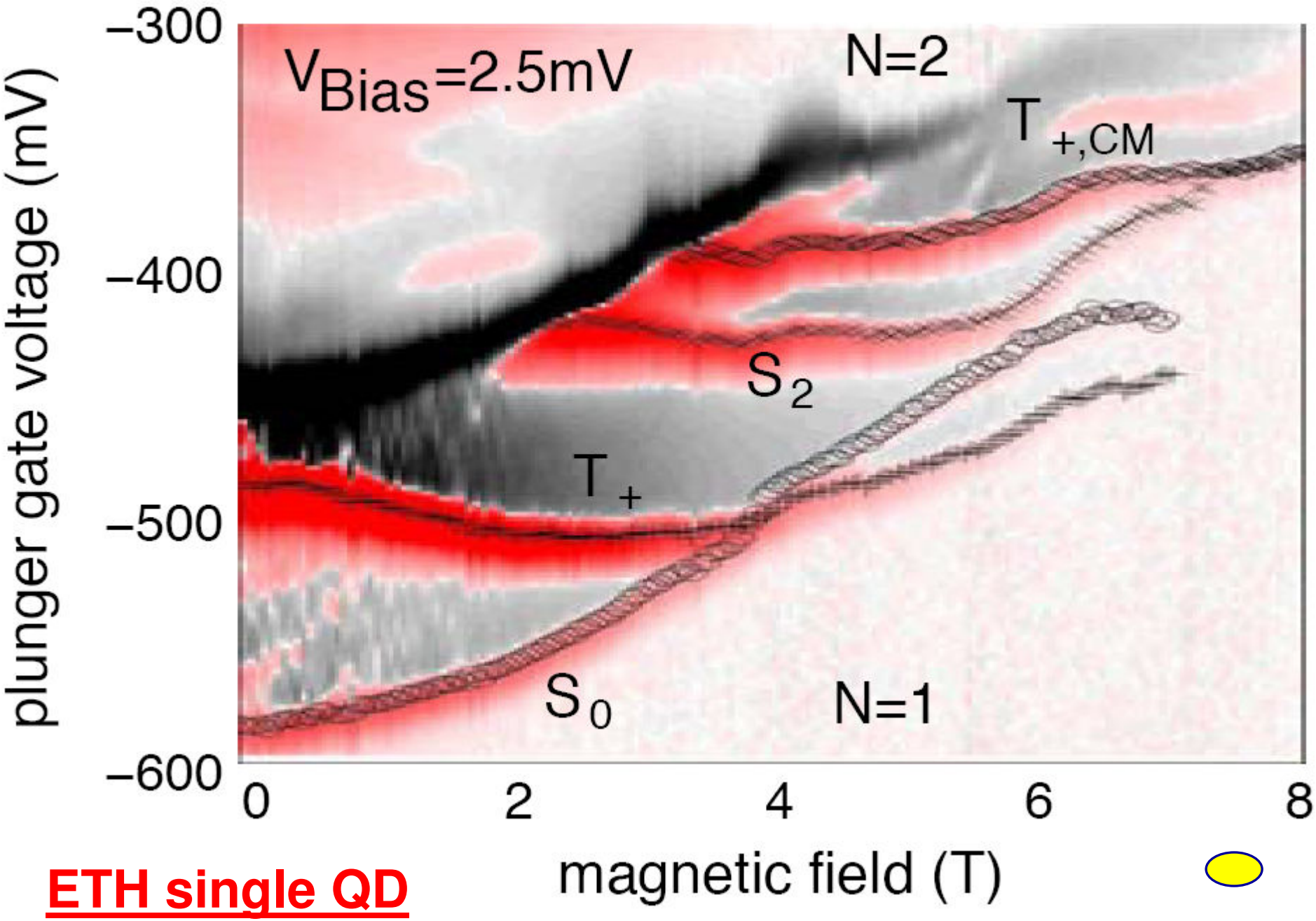
N=2e

η= 0.72

η=ratio of principal axes

Single QD ETH Zurich (K. Ensslin, Th. Ihn…)

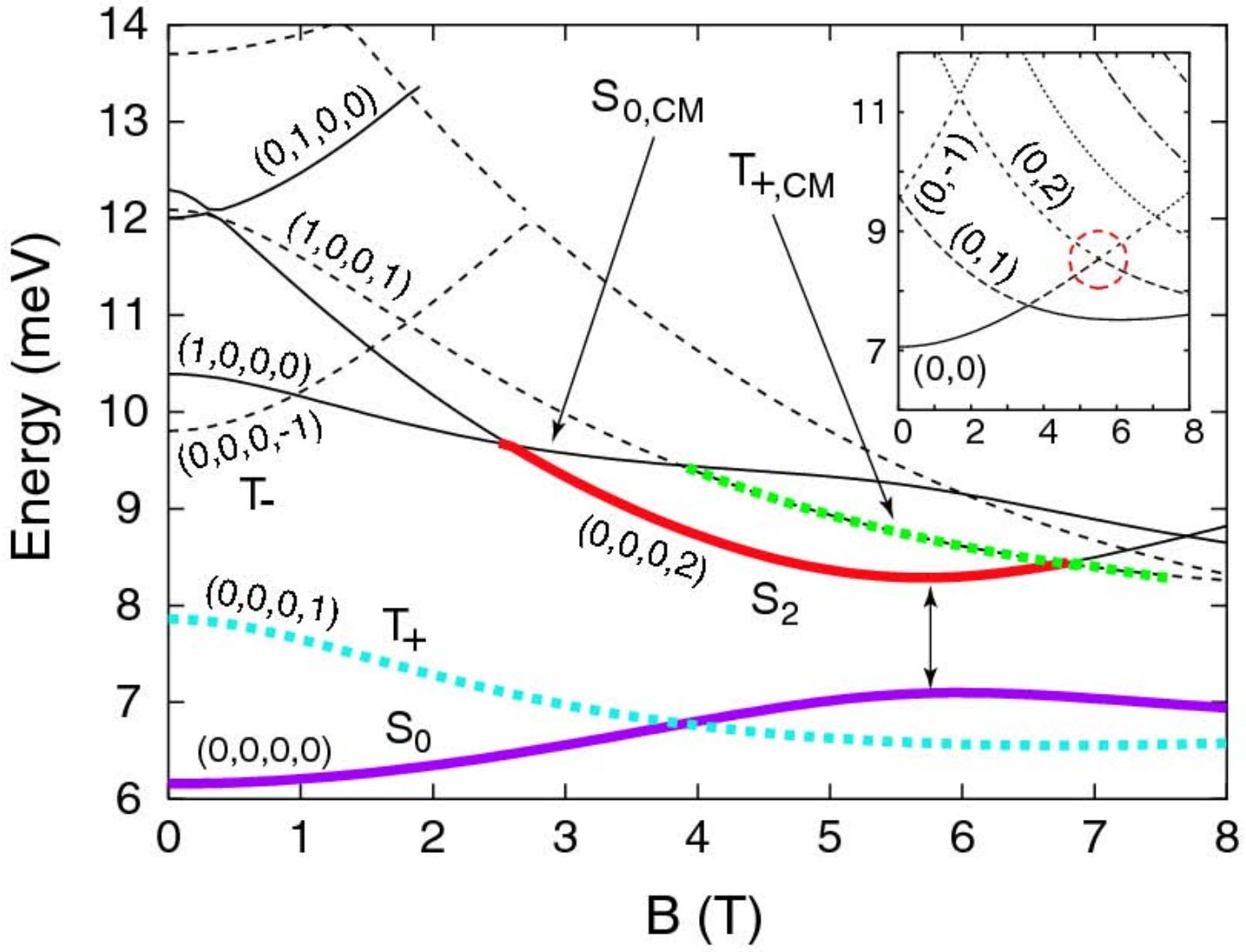




ETH single QD

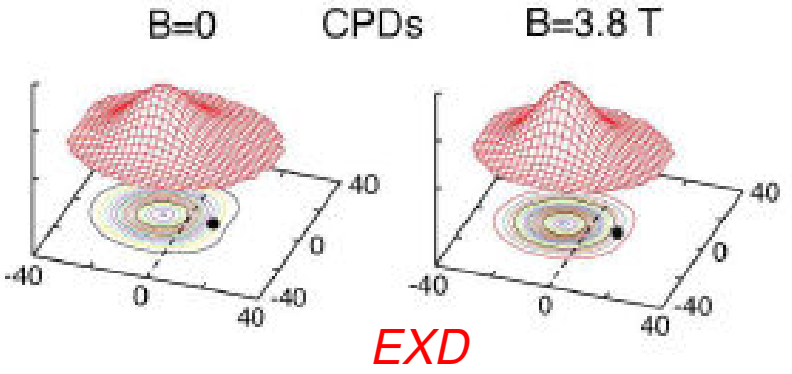
ETH single QD

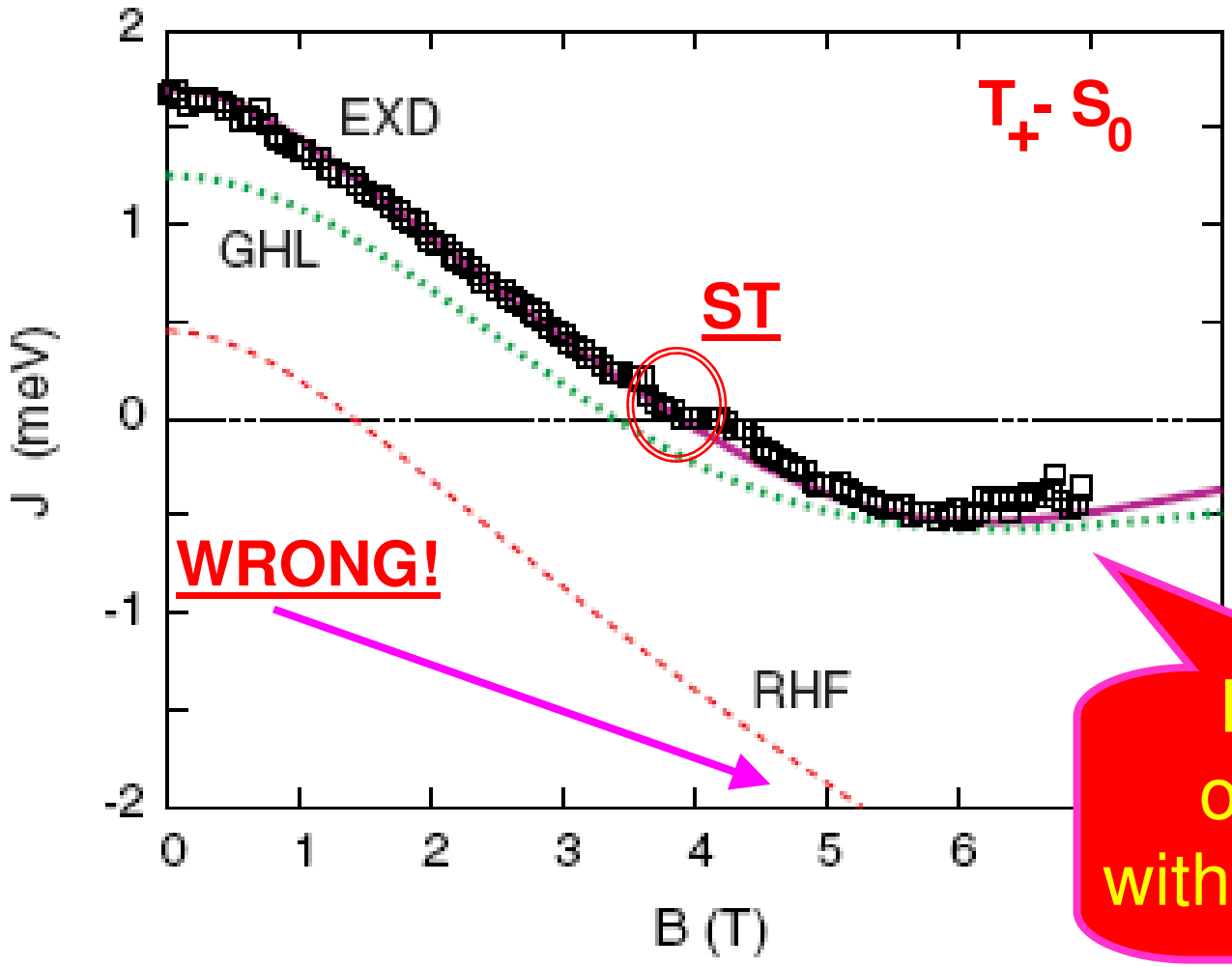
EXD = Exact diagonalization



ETH single QD

hwx=4.23 meV; hwy=5.84 meV;
m*=0.070; κ=12.5; γ=0.86





Genelarized Heitler-London wave function

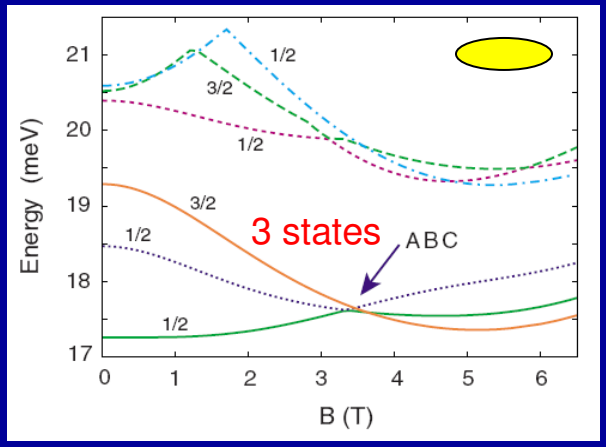
Dissociation of the 2e WM within the single QD

N=3e

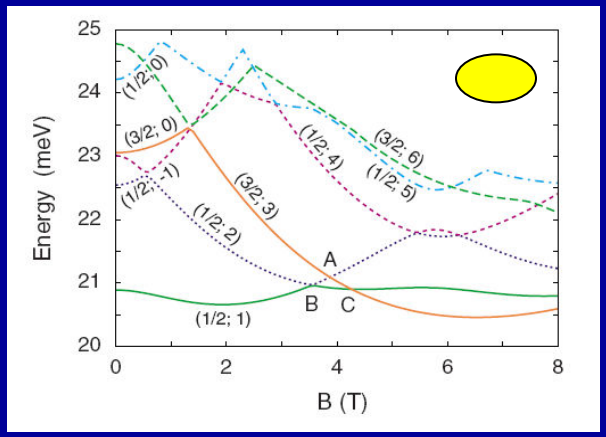
Excitation spectra

Yuesong Li et al., PRB **76**, 245310 (2007)

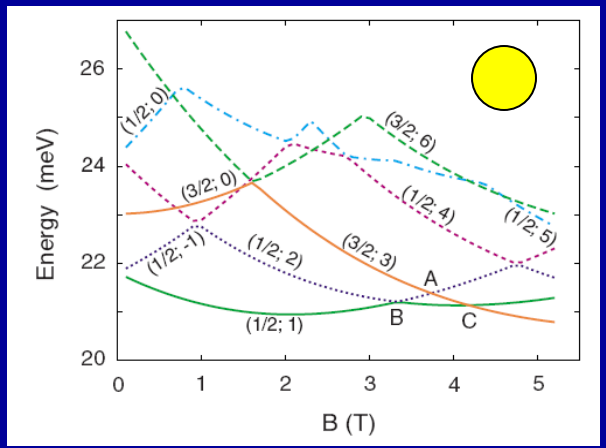
κ=12.5
η=1/2



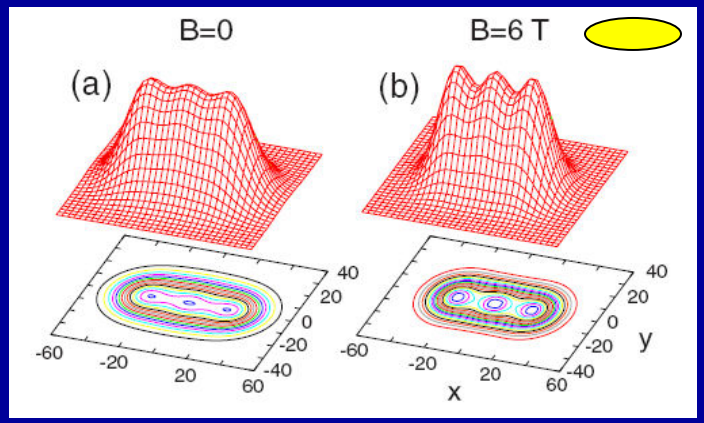
κ=12.5
η=0.72



κ=12.5
η=1



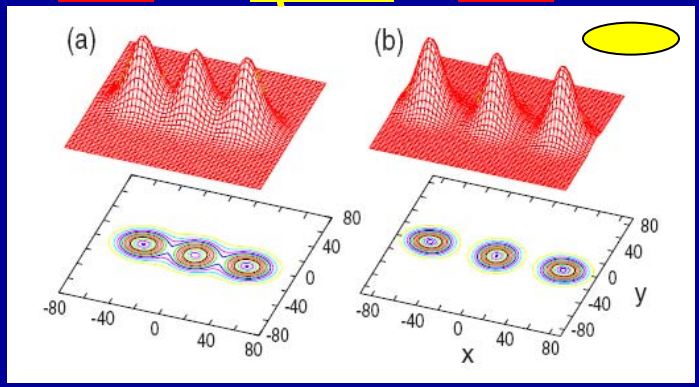
κ=12.5 η=1/2



Electron densities

Pinned Wigner Molecule

κ=3 η=1/2 κ=1



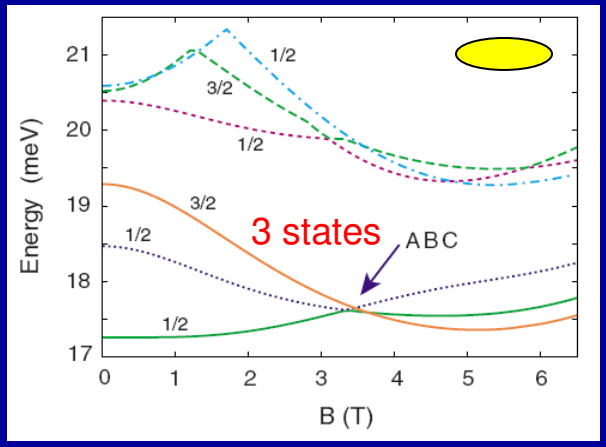
N=3e

Excitation spectra

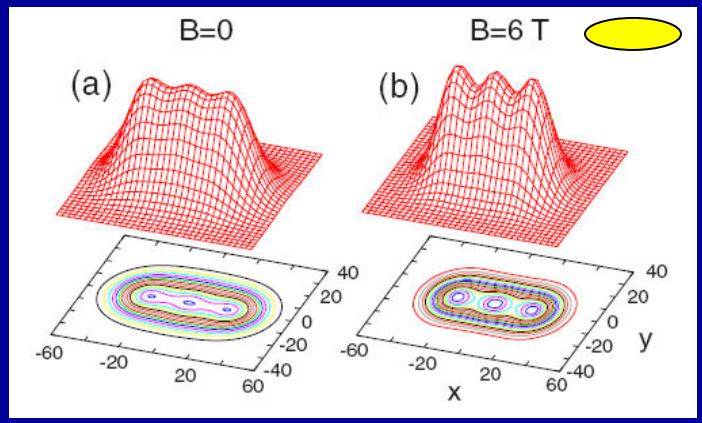
Yuesong Li et al., PRB **76**, 245310 (2007)

κ=12.5

η=1/2

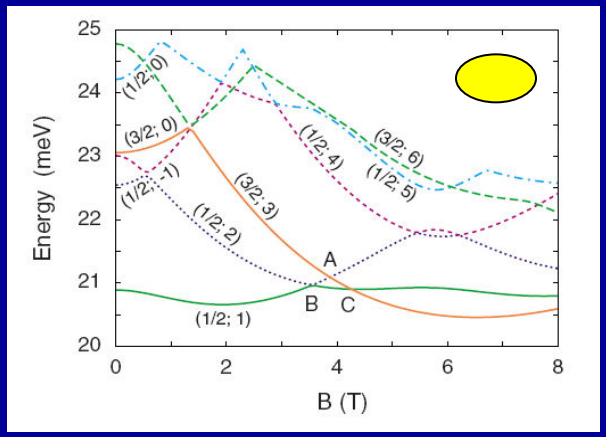


κ=12.5 **η=1/2**



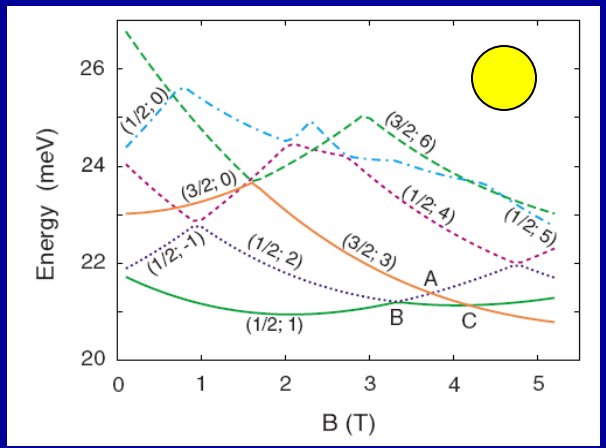
κ=12.5

η=0.72



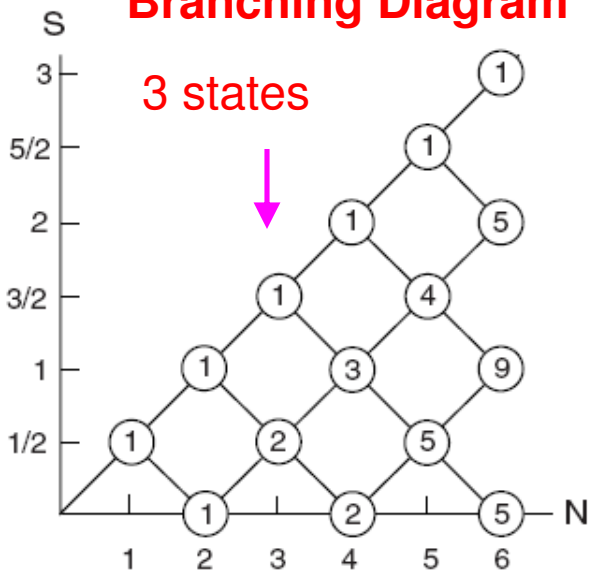
κ=12.5

η=1



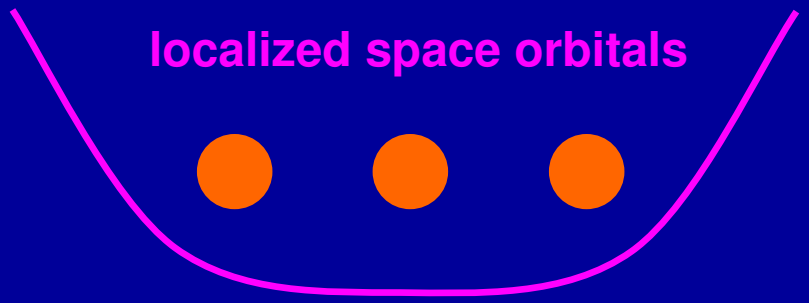
Electron densities

Branching Diagram



Formation of three-electron Wigner molecule

Elliptic QD



$\alpha|\uparrow\downarrow\uparrow\rangle+\beta\,|\downarrow\uparrow\uparrow\rangle+\gamma\,|\uparrow\uparrow\downarrow\rangle$

Entangled three-qubit W-states

1) $\alpha=2,\ \beta=-1,\ \gamma=-1 \Rightarrow (1/2,1/2;\ 1)$

2) $\alpha=0,\ \beta=1,\ \gamma=-1 \Rightarrow (1/2,1/2;\ 2)$

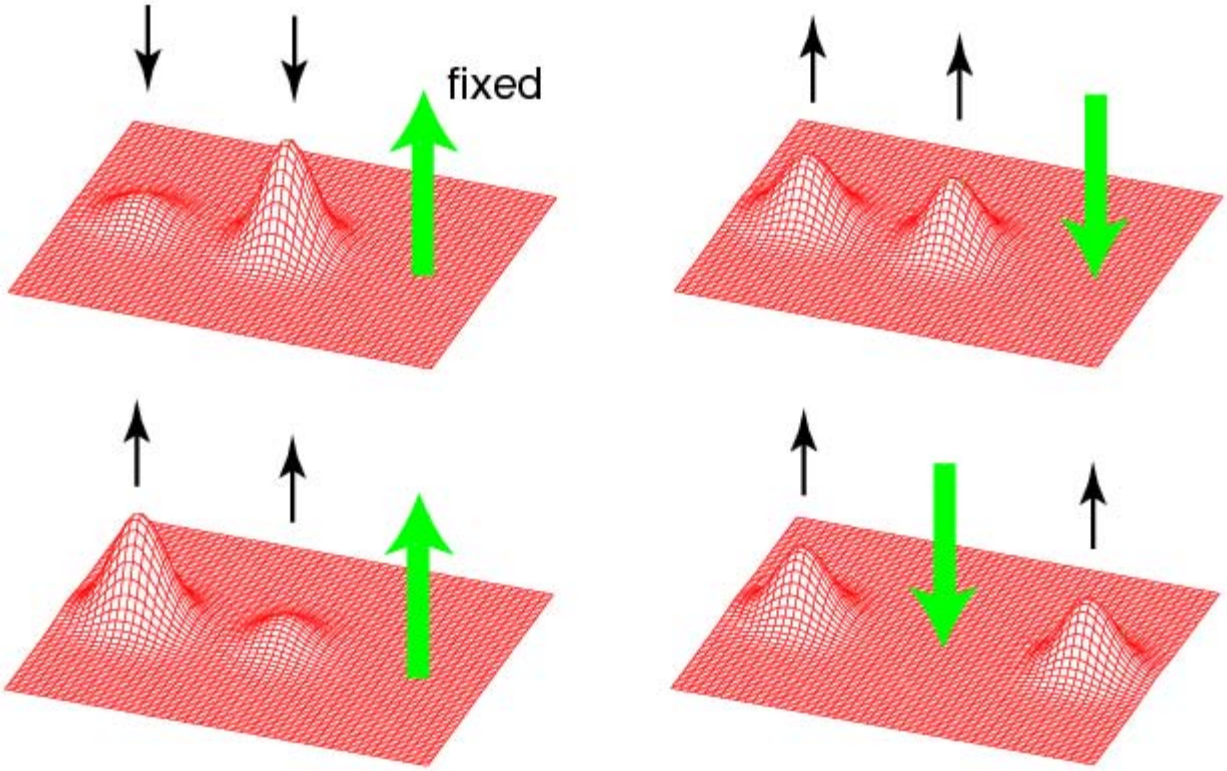
3) $\alpha=\beta=\gamma=1 \Rightarrow (3/2,1/2)$

Study entanglement by using

Spin resolved CPDs for EXD wfs

Ground-state (1/2,1/2); hwx=3.137 meV; hwx/hwy=1/2;

$m^*=0.067m_e$; B=0; κ=1



EXD wf ~ $2|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle$

Control and measurement of three-qubit entangled states

C. F. Roos[1], Mark Riebe[1], H. Häffner[1], W. Hänsel[1], J. Benhelm[1], G. P. T. Lancaster[1], C. Becher[1], F. Schmidt-Kaler[1] & R. Blatt[1,2]

[1]Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
[2]Institut für Quantenoptik und Quanteninformation, Östereichische Akademie der Wissenschaften

- Basics of ion trap quantum computers
- Entangling operations (Bell states, CNOT)
- Generation of W- and GHZ-states
- Selective read-out of a quantum register
- Entanglement transformation by condional operations

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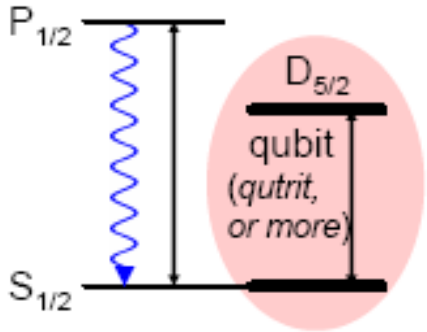
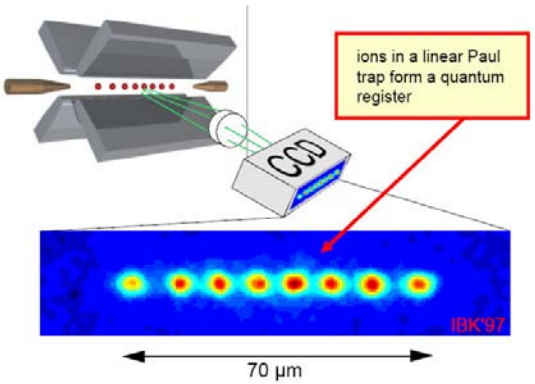
AG Quantenoptik und Spektroskopie

Qubits with trapped ions



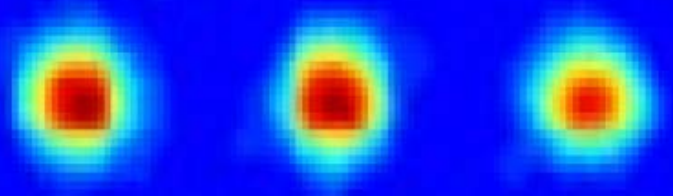
S – D transitions in alkaline earths:
Ca^+, Sr^+, Ba^+, Ra^+, (Yb^+, Hg^+) etc.

The system: String of $^{40}Ca^+$ ions in a linear Paul trap



Entangled states with three ions

GHZ-states: $|SSS + DDD\rangle$

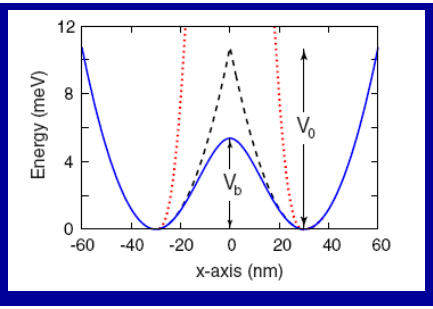


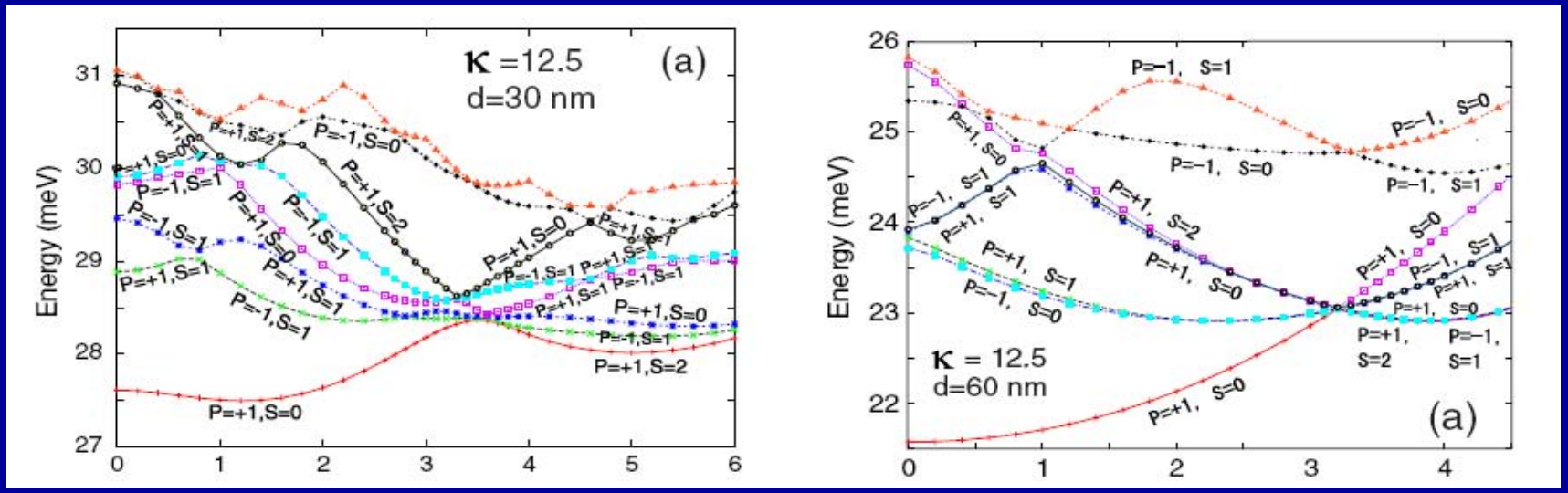
W-states: $|SDD + DSD + DDS\rangle$

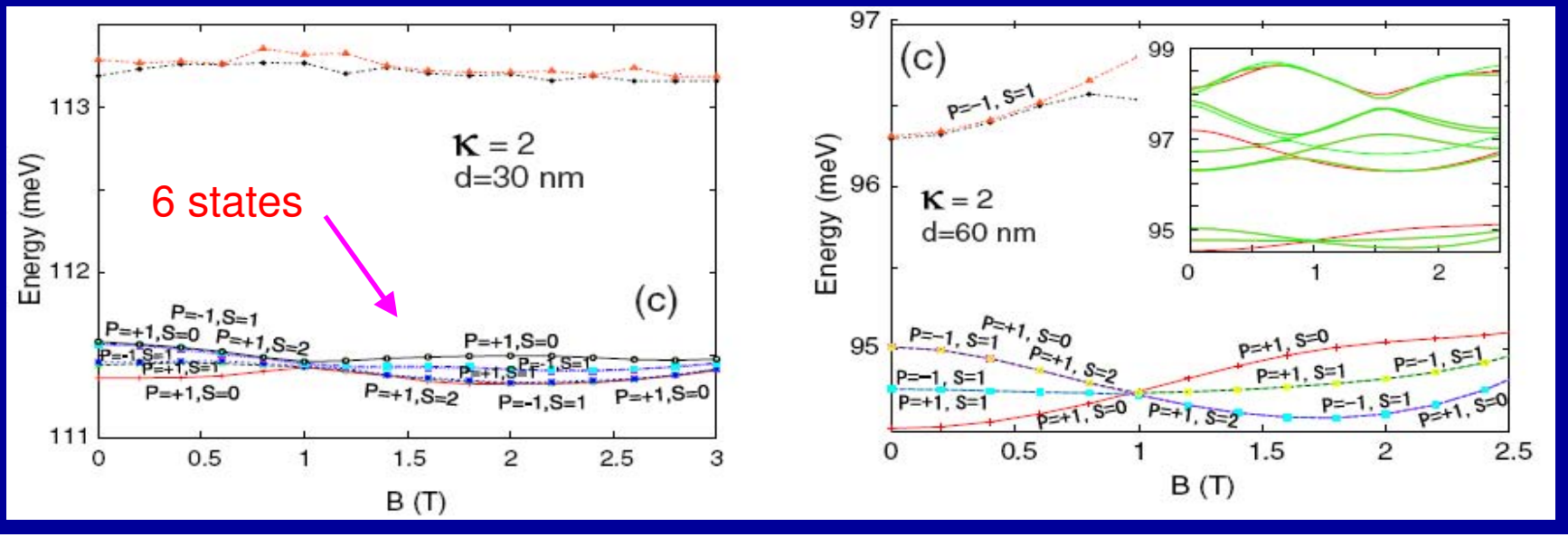
Quantum Dot Helium Molecule

N=4e Ying Li et al.: PRB 80, 045326 (2009)

EXD calculation







Quantum Dot Helium Molecule

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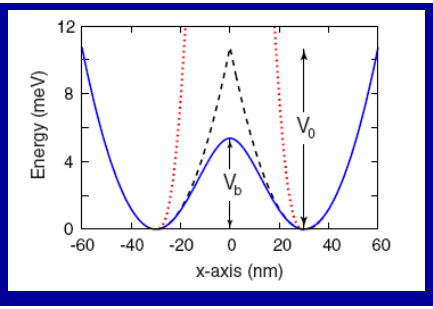
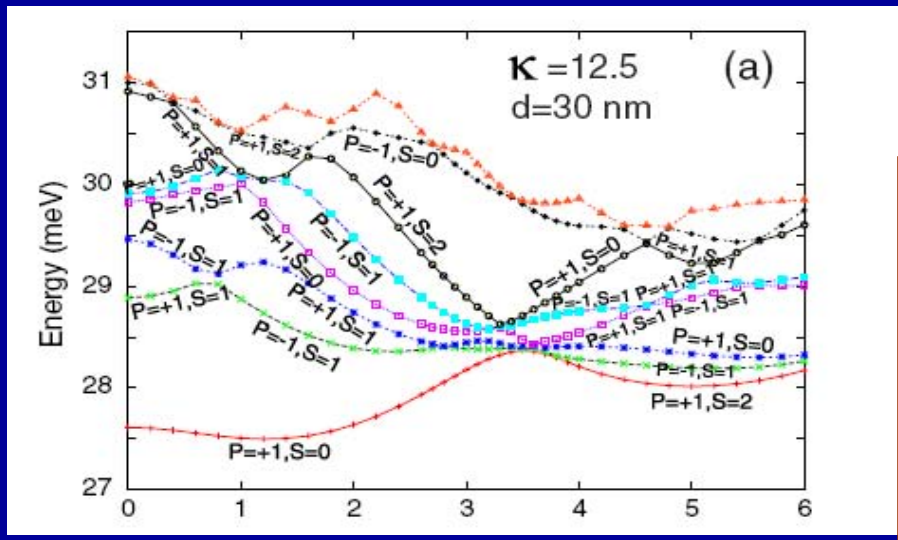
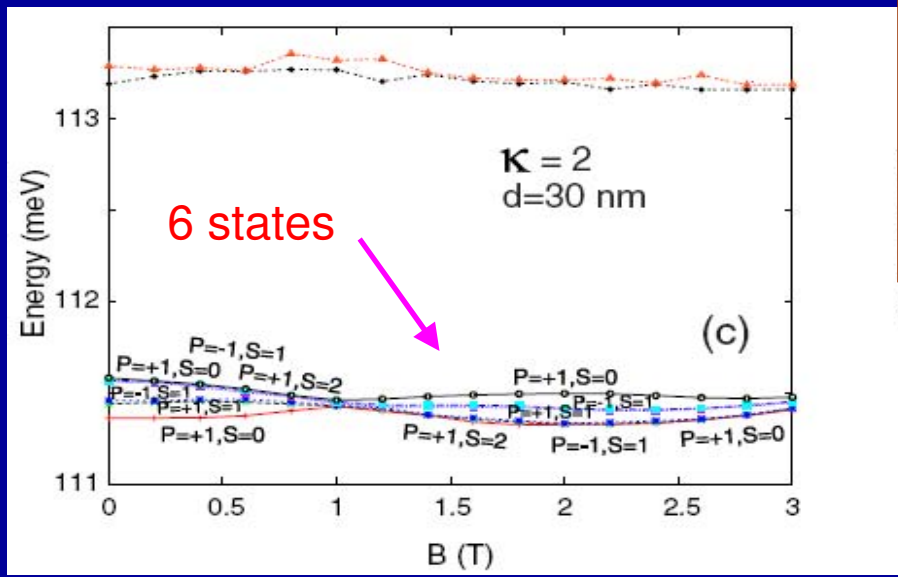
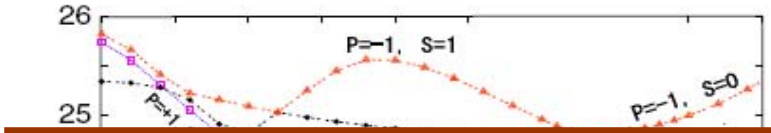
EXD calculation

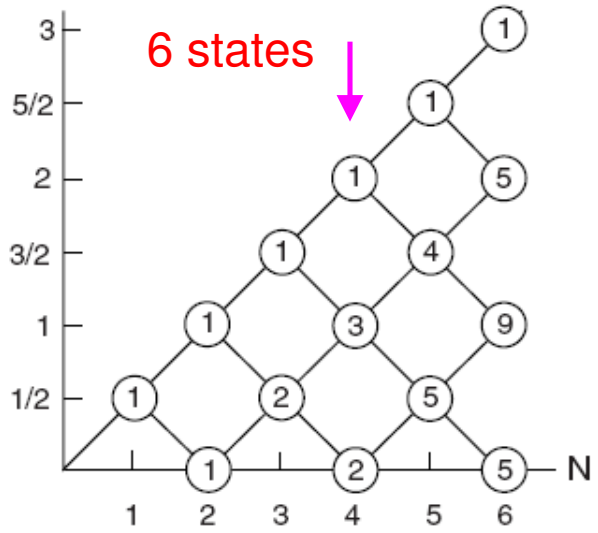


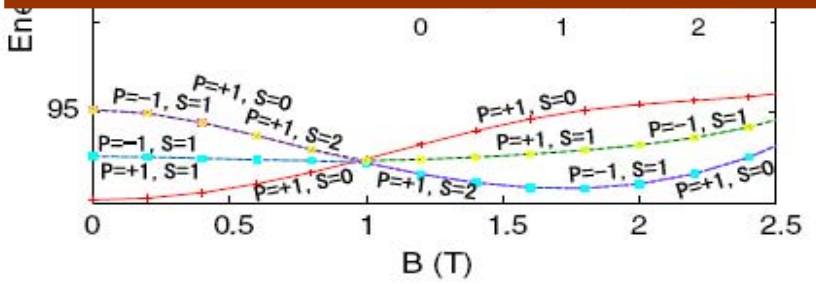






Branching Diagram

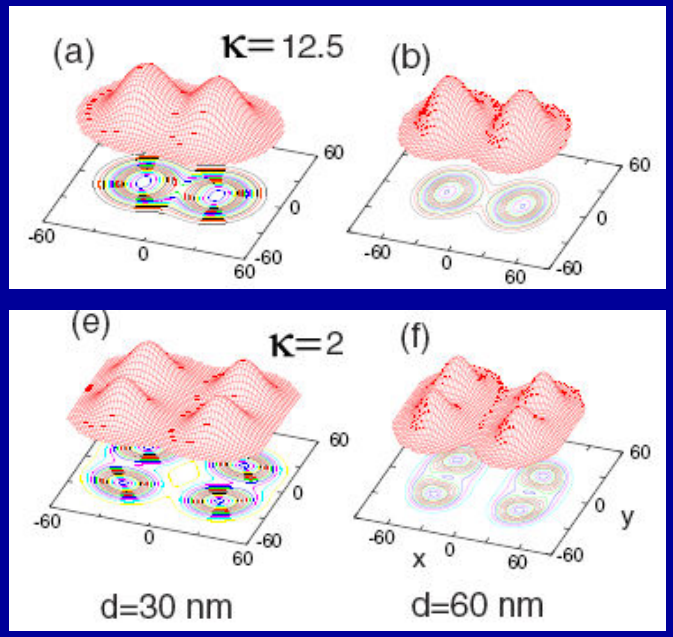




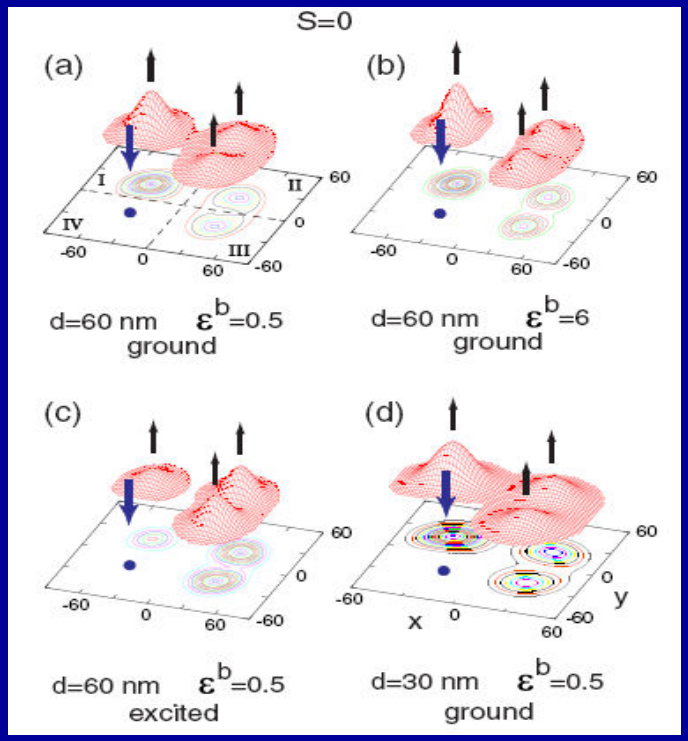
States at B=0

$S = 0, S_z = 0$

Ground State Electron Densities



Spin-resolved Pair Correlations



κ=2

$$|\Psi_N^{\mathrm{EXD}}(S, S_z; k)\rangle = \sum_I C_I^N(S, S_z; k)|SD(I; N, S_z)\rangle$$

I ~ 100,000

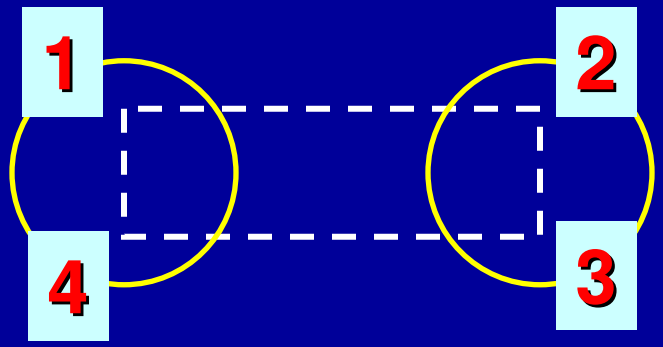
Slater Determinant

→ **Spin functions**

$$\chi_{00}^{(1)} = -\frac{1}{2}|\uparrow\uparrow\downarrow\downarrow\rangle + \frac{1}{2}|\uparrow\downarrow\uparrow\downarrow\rangle + \frac{1}{2}|\downarrow\uparrow\downarrow\uparrow\rangle - \frac{1}{2}|\downarrow\downarrow\uparrow\uparrow\rangle$$

$$\chi_{00}^{(2)} = \frac{1}{2\sqrt{3}}|\uparrow\uparrow\downarrow\downarrow\rangle + \frac{1}{2\sqrt{3}}|\uparrow\downarrow\uparrow\downarrow\rangle - \frac{1}{\sqrt{3}}|\uparrow\downarrow\downarrow\uparrow\rangle - \frac{1}{\sqrt{3}}|\downarrow\uparrow\uparrow\downarrow\rangle + \frac{1}{2\sqrt{3}}|\downarrow\uparrow\downarrow\uparrow\rangle + \frac{1}{2\sqrt{3}}|\downarrow\downarrow\uparrow\uparrow\rangle$$

4-site Heisenberg cluster



$$\mathcal{H}_H(B) = \tilde{J}_{12}(B)(\mathbf{S}_1\cdot\mathbf{S}_2 + \mathbf{S}_3\cdot\mathbf{S}_4) + \tilde{J}_{14}(B)(\mathbf{S}_1\cdot\mathbf{S}_4 + \mathbf{S}_2\cdot\mathbf{S}_3)$$

$$S_z = 0$$

$$\frac{1}{2}\begin{pmatrix}
\tilde{J}_{12}-\tilde{J}_{14} & \tilde{J}_{14} & 0 & 0 & \tilde{J}_{14} & 0 \\
\tilde{J}_{14} & -(\tilde{J}_{12}+\tilde{J}_{14}) & \tilde{J}_{12} & \tilde{J}_{12} & 0 & \tilde{J}_{14} \\
0 & \tilde{J}_{12} & \tilde{J}_{14}-\tilde{J}_{12} & 0 & \tilde{J}_{12} & 0 \\
0 & \tilde{J}_{12} & 0 & \tilde{J}_{14}-\tilde{J}_{12} & \tilde{J}_{12} & 0 \\
\tilde{J}_{14} & 0 & \tilde{J}_{12} & \tilde{J}_{12} & -(\tilde{J}_{12}+\tilde{J}_{14}) & \tilde{J}_{14} \\
0 & \tilde{J}_{14} & 0 & 0 & \tilde{J}_{14} & \tilde{J}_{12}-\tilde{J}_{14}
\end{pmatrix}$$

$|1\rangle \rightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$ $|2\rangle \rightarrow |\uparrow\downarrow\uparrow\downarrow\rangle$ ..... $|6\rangle \rightarrow |\downarrow\downarrow\uparrow\uparrow\rangle$

4-site Heisenberg cluster: energies and eigenvectors

$$\mathcal{E}_1 = -(\tilde{J}_{14} + \tilde{J}_{12})/2,$$

$$\mathcal{E}_2 = (\tilde{J}_{14} - \tilde{J}_{12})/2,$$

$$\mathcal{E}_3 = (\tilde{J}_{12} - \tilde{J}_{14})/2,$$

$$\mathcal{E}_4 = (\tilde{J}_{14} + \tilde{J}_{12})/2,$$

$$\mathcal{E}_5 = -(\tilde{J}_{14} + \tilde{J}_{12})/2 - Q(\tilde{J}_{14}, \tilde{J}_{12}),$$

$$\mathcal{E}_6 = -(\tilde{J}_{14} + \tilde{J}_{12})/2 + Q(\tilde{J}_{14}, \tilde{J}_{12}),$$

where

$$Q(a,b) = \sqrt{a^2 - ab + b^2}.$$

Explain EXD spectra

$$\mathcal{V}_1 = \{0,-1,0,0,1,0\}, \quad S = 1,$$

$$\mathcal{V}_2 = \{0,0,-1,1,0,0\}, \quad S = 1,$$

$$\mathcal{V}_3 = \{-1,0,0,0,0,1\}, \quad S = 1,$$

$$\mathcal{V}_4 = \{1,1,1,1,1,1\}, \quad S = 2,$$

$$\mathcal{V}_5 = \{1,-\mathcal{X},-1+\mathcal{X},-1+\mathcal{X},-\mathcal{X},1\}, \quad S = 0,$$

$$\mathcal{V}_6 = \{1,-\mathcal{Y},-1+\mathcal{Y},-1+\mathcal{Y},-\mathcal{Y},1\}, \quad S = 0,$$

where

$$\mathcal{X} = r + Q(1,r),$$

$$\mathcal{Y} = r - Q(1,r),$$

and $r = \tilde{J}_{12}/\tilde{J}_{14}$.

$\mathcal{V}_5 \to \{1,-1,0,0,-1,1\}$ $\mathcal{V}_6 \to \{1,1,-2,-2,1,1\}$

Agree with EXD spin functions

SUMMARY

Under appropriate conditions, 2D electrons in anisotropic single and double quantum dots do localize, forming pinned Wigner Molecules (PWMs)

The excitation spectra and spin functions of PWMs can be understood via finite-Heisenberg-cluster Hamiltonians with B-dependent exchange constants J(B).

Spin functions are associated with classes of well known strongly entangled states, e.g., W-states, Dicke states, etc…

Signatures of the Heisenberg-type spectra do survive even for weaker localization (corresponding to current available experimental lateral quantum dots).